Integrator-based circuit-independent chaotic oscillator structure

A. S. Elwakil[a)]
Department of Electrical and Computer Engineering, Faculty of Engineering, University of Sharjah, P.O. Box 27272, Sharjah, United Arab Emirates

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An integrator-based chaotic oscillator structure composed of three cascaded inverting, noninverting, and differential integrators is presented. The nonlinearity responsible for folding the trajectories is introduced by a single switching diode which is controlled by the output of the first integrator in the cascade. Chaotic behavior is verified on the functional level of the structure rendering it circuit-independent. A possible circuit realization is given and a canonical single-parameter-controlled ordinary differential equation capturing the qualitative dynamics of similar fourth-order integrator-based chaotic oscillators is proposed.
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The idea of being able to intentionally design an electronic circuit to produce chaotic signals has motivated a research flux over the past few years. In this article, a simple architecture for a chaotic oscillator, which can incorporate a wide variety of circuit realizations, is presented.

I. INTRODUCTION

The topic of chaotic oscillator design has received significant attention during the past few years (see, for example, Refs. 1–4). On the one hand, some researchers were motivated by the promising applications of chaos, particularly in communication systems.[5,6] On the other hand, other researchers were more interested in resolving the mystery of chaotic oscillator circuit design such that techniques developed under the linear circuit theory of design can be used by conventional designers to realize systematically these oscillators.[1–4] Pronounced advances in this direction have particularly identified a robust kernel (core engine) upon which autonomous chaos generators are constructed.[4] Systematic modification procedures carried-out on this engine automatically result in chaos being generated.

In Refs. 4 and 7, an important step forward was taken; that is to raise the level of chaotic oscillator design from the circuit-specific level to the more general circuit-structure level. For this purpose, circuit-independent models for several classes of chaotic oscillators were introduced. It was identified that the function of the core engine should be to produce unstable sinusoidal oscillations. Hence, the simplest possible structures for this engine were modelled in a novel manner, which is independent of any particular circuit realization, in Ref. 4. The same models were also used in Ref. 7 to obtain circuit-independent RC realizations of Chua's circuit.

It is the aim of this work to follow up in the same direction by proposing a novel chaotic oscillator structure based on cascaded integrators. No circuit-specific detail is required in order to verify the chaotic nature of the structure, which also incorporates a signal diode as its only source of nonlinearity. By moving to this functional behavior design level, freedom is given to designers to choose their own active building blocks and suitable methods to perform integration according to the specifications of their target applications. A circuit based on current feedback op amps is given as one possible realization of the structure. Finally, we propose a canonical model, which is controlled via a single parameter, to capture the qualitative dynamics of a large class of low-dimensional fourth-order integrator-based chaotic oscillators.

II. INTEGRATOR-BASED STRUCTURE

Consider the configuration shown in Fig. 1, where three integrators are cascaded in a feedback loop. All integrators are modelled by linear first order transfer functions of the form $T(s)=C/D(s)$; C is constant and s is the complex frequency $j\omega$. The first integrator is noninverting and has a time constant T_1, whereas the second integrator is inverting and has a time constant T_2; both integrators are lossless. The third stage in the cascade is a lossy differential integrator with time constant T_1 and gain K. A signal diode in series with its current-limiting resistor are inserted at the output of the first integrator V_{O1}. Thus, the diode switches on and off according to the voltage developed at this node. However, in order to model accurately the switching action of the diode, the parasitic transit capacitance C_D (see Fig. 1), which ap-

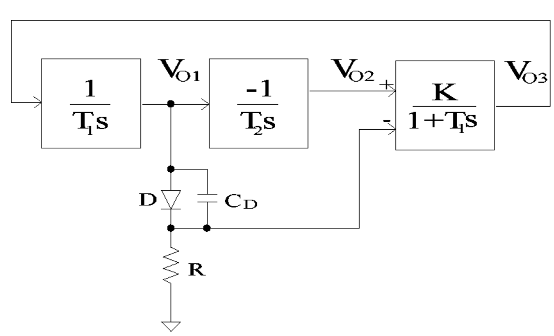

FIG. 1. Proposed integrator-based chaotic oscillator structure.

a)Electronic mail: elwakil@sharjah.ac.ae

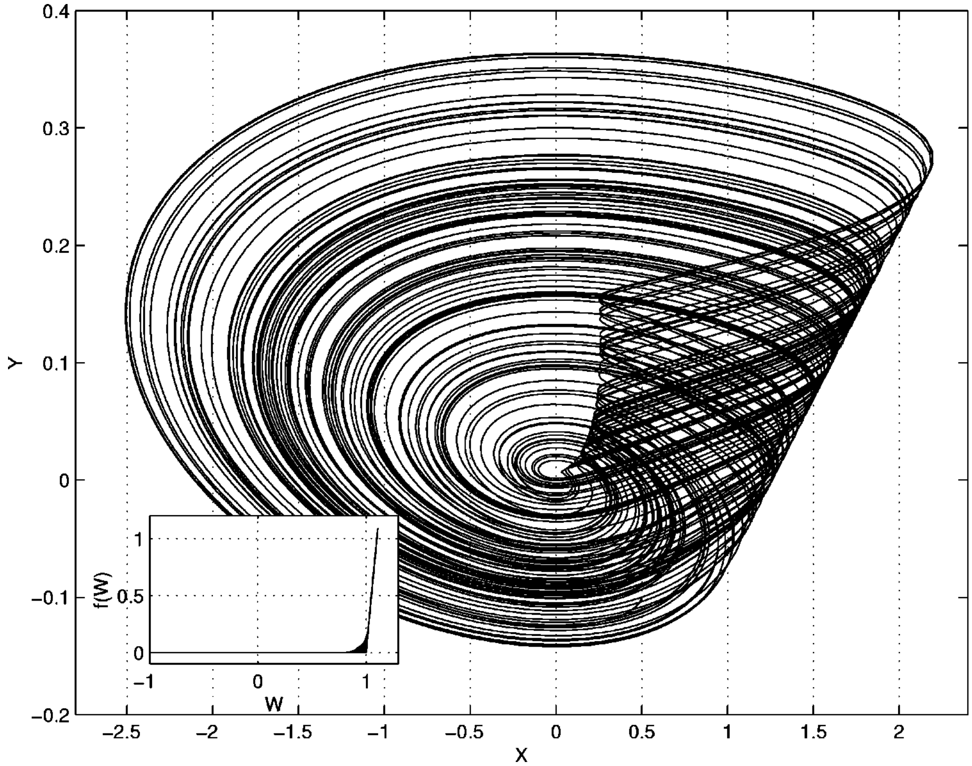

FIG. 2. Chaotic attractor in the $X-Y$ plane ($a=0.1$, $\epsilon=0.01$, $K=18$, $K_D=10$).

pears across the diode, has to be considered.[8] Hence, the structure is described by the following set of equations:

$$T_1\dot{V}_{O1}=V_{O3}, \tag{1a}$$

$$T_2\dot{V}_{O2}=-V_{O1}, \tag{1b}$$

$$T_1\dot{V}_{O3}=K(V_{O2}-V_{O1}+V_{CD})-V_{O3}, \tag{1c}$$

$$C_D\dot{V}_{CD}=\frac{V_{O1}-V_{CD}}{R}-I_N, \tag{1d}$$

and I_N is the nonlinear diode current modelled in piecewise-linear form by

$$I_N=\frac{1}{R_D}\begin{cases} V_{CD}-V_\gamma & V_{CD}\geqslant V_\gamma, \\ 0 & V_{CD}<V_\gamma. \end{cases} \tag{2}$$

R_D and V_γ are diode forward conduction resistance and voltage drop, respectively.

By introducing the following variables: $X=V_{O1}/V_\gamma$, $Y=V_{O2}/V_\gamma$, $Z=V_{O3}/V_\gamma$, $W=V_{CD}/V_\gamma$, $a=T_1/T_2$, $\epsilon=RC_D/T_1$, $K_D=R/R_D$ and normalizing time with respect to T_1, the dimensionless form of (1) becomes

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \epsilon\dot{W} \end{pmatrix}=\begin{pmatrix} 0 & 0 & 1 & 0 \\ -a & 0 & 0 & 0 \\ -K & K & -1 & K \\ 1 & 0 & 0 & -1-b \end{pmatrix}\begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix}+\begin{pmatrix} 0 \\ 0 \\ 0 \\ b \end{pmatrix} \tag{3a}$$

and

$$b=\begin{cases} K_D & W\geqslant 1, \\ 0 & W<1. \end{cases} \tag{3b}$$

We have numerically integrated this model using an adaptive step Runge–Kutta algorithm taking $K=18$, $K_D=10$, $a=0.1$, and $\epsilon=0.01$. The observed chaotic attractor projection in the $X-Y$ plane is shown in Fig. 2. Note that the chaotic

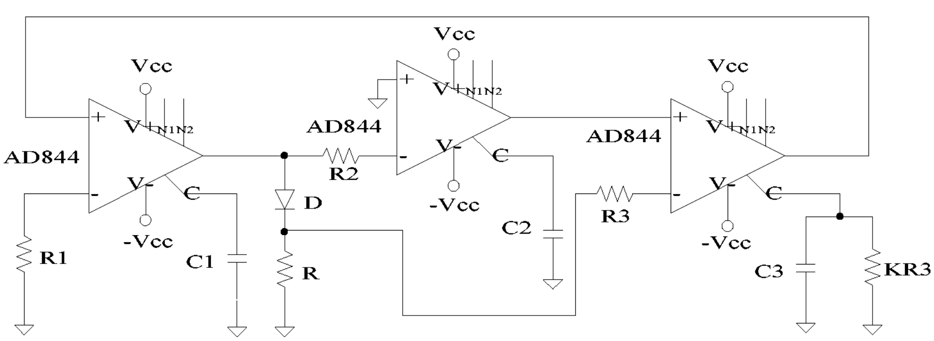

FIG. 3. Possible implementation of the proposed structure using AD844 current feedback operational amplifiers.

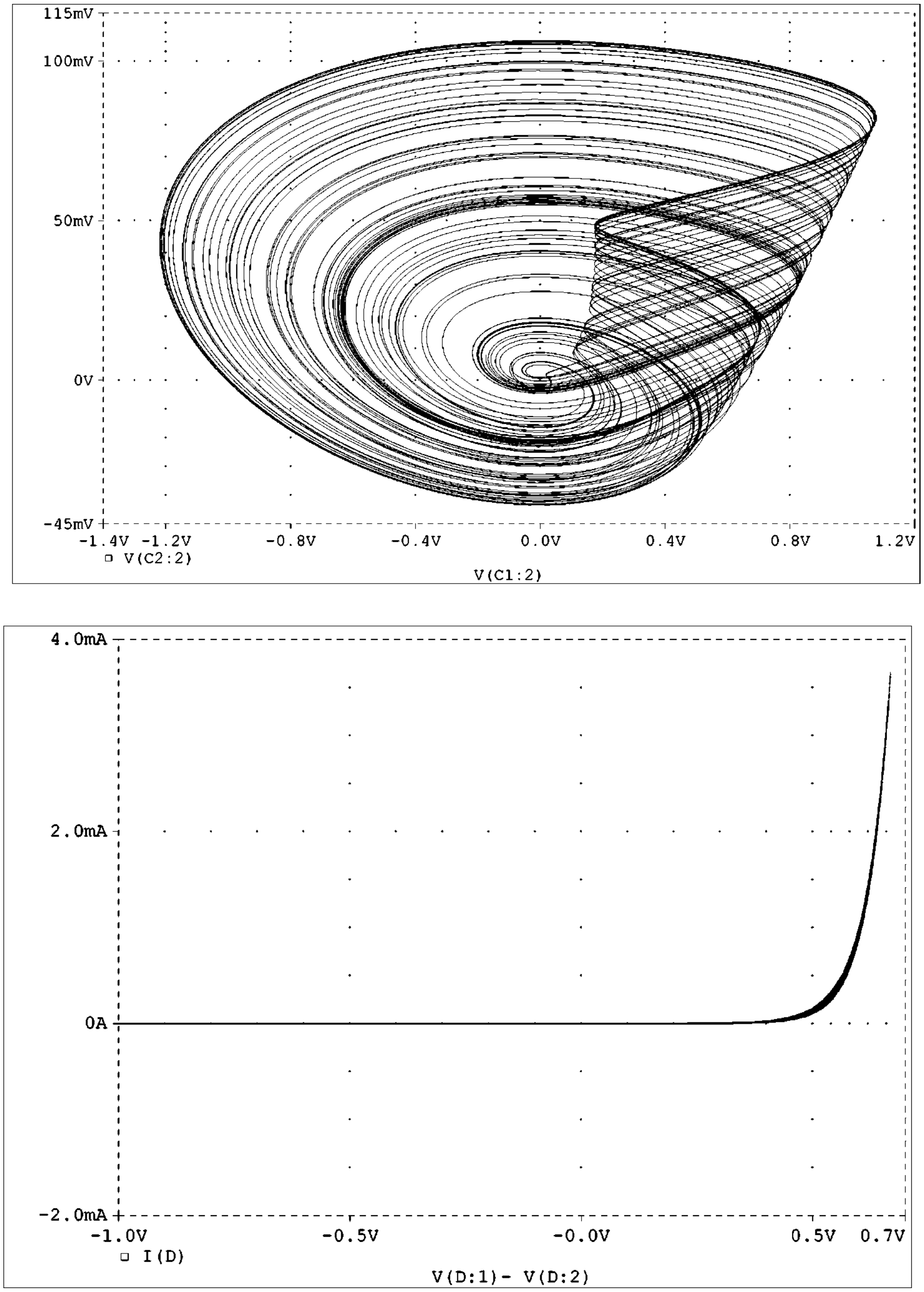

FIG. 4. (a) Pspice simulation of the $V_{C1}-V_{C2}$ projection of the chaotic attractor from the circuit implementation of Fig. 3, and (b) dynamic $I-V$ characteristics of the diode.

attractor is effectively living in the three-dimensional $X-Y-Z$ subspace under the limit condition $\epsilon\rightarrow 0$, which reflects the parasitic effect of C_D. From the data series of X, a single positive Lyapunov exponent equal to 0.0072 was calculated.

It is clear from (3) that the core engine of the system is a quadrature sinusoidal oscillator in the $X-Y$ plane ($Z=W=0$) with a fundamental frequency of oscillation $\omega_0=\sqrt{a}$. Therefore, the smaller the value of a, i.e., smaller ratio of the time constants T_1/T_2, the more time spent by the system in the $X-Y$ plane before evolving into a higher dimensional space. The system described by (3) has a single equilibrium point at the origin $(x_0,y_0,z_0,w_0)=(0,0,0,0)$ which is real and unstable in the region $W<1$ while it is virtual and stable in the region $W\geqslant 1$. [A virtual equilibrium point is one which lies physically outside its corresponding subspace ($W\geqslant 1$ in this case).] The calculated eigenvalues corresponding to Fig. 2 are: $(-0.1,-10^4,-0.45\pm j4.19)$ when $b=K_D$ and $(-1.76,-99.8,0.29\pm j0.97)$ when $b=0$.

Since each integrator in Fig. 1 is represented only by its input–output transfer function, it is possible to obtain numerous circuit realizations. It is important to note that the lossy differential integrator can be considered as a differential amplifier with gain K followed by a passive low-pass filter. Also, the phase inversion performed by the inverting integrator can be delayed to the following stage, i.e., the differential integrator. In this case, the first two stages are identical non-

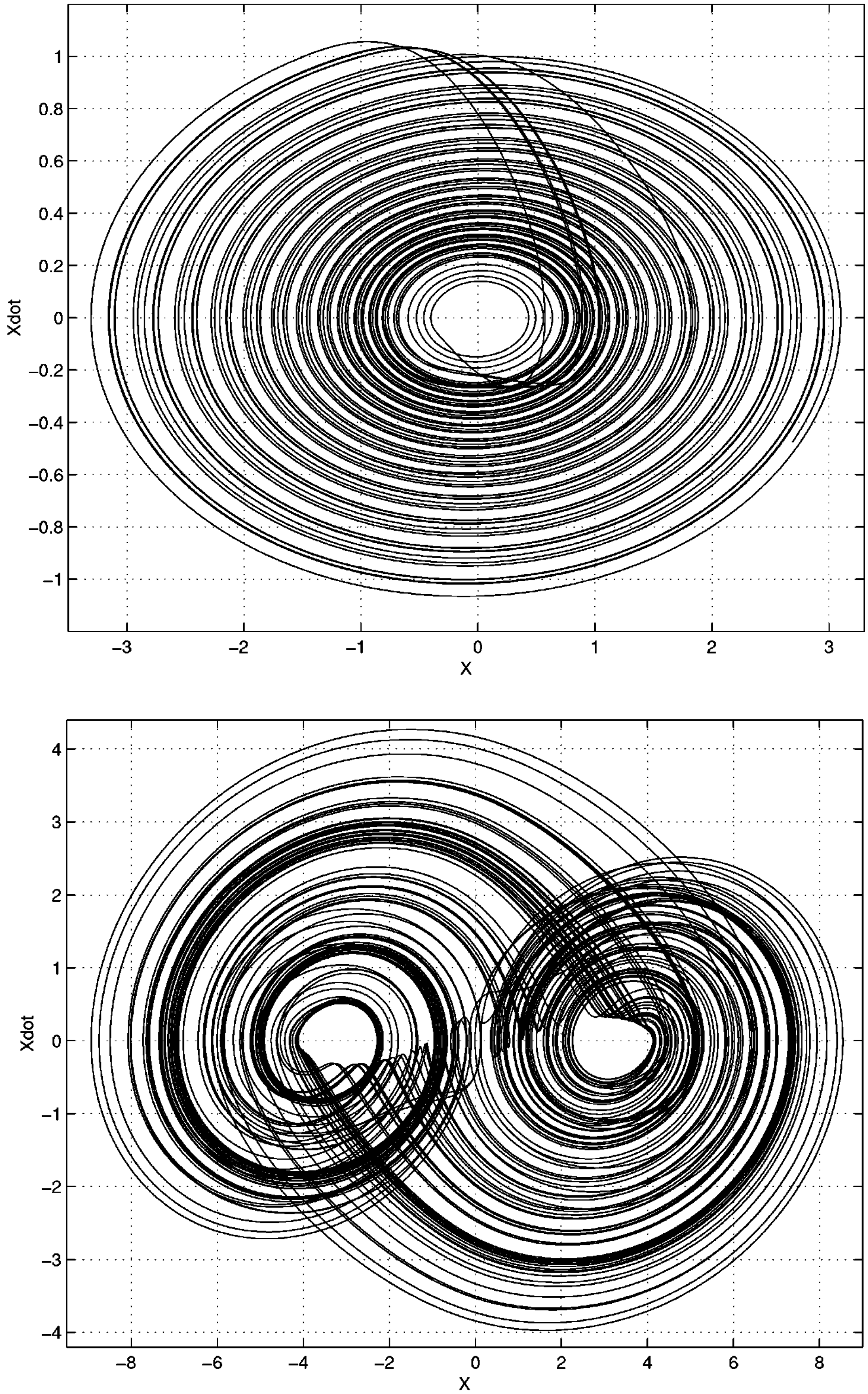

FIG. 5. Chaotic attractors of the proposed canonical model in the $X-\dot{X}$ plane. (a) f is antisymmetrical and $\epsilon=0.1$, (b) f is symmetrical and $\epsilon=0.25$.

inverting integrators with different time constants whereas the last stage becomes an inverting summer-amplifier with gain $-K$ followed by a passive low-pass filter. This possibility of reorganizing the transfer functions adds even more flexibility to the circuit-design process.

To verify the performance of the proposed structure, we have chosen to synthesize a circuit where all integrators are implemented using current feedback op amps (CFOAs).[9] The circuit is shown in Fig. 3 and has the advantage of all grounded capacitors.

A Pspice transient simulation of the circuit was performed taking $R_1=R_3=1\ \mathrm{k}\Omega$, $C_1=C_3=0.5$ nF, $R_2=5\ \mathrm{k}\Omega$, $C_2=10$ nF, $R=100\ \Omega$, and $K=5.5$. A general purpose D1N4148 diode was chosen and the AD844 CFOAs biased with ± 9 V supplies were used. The observed $V_{C1}-V_{C2}$ phase-space trajectory is shown in Fig. 4(a) and is seen to compare well with the $X-Y$ projection in Fig. 2. Figure 4(b) represents the diode characteristics dynamically plotted while Spice performs transient simulations. Of course, the diode model used by Spice is more comprehensive than the simple model of (2) which is plotted for comparison with Fig. 4(b) in the lower left corner of Fig. 2.

III. SINGLE-PARAMETER CANONICAL MODEL

The system described by (3) is a fourth-order system which collapses into a three-dimensional subspace as $\epsilon \rightarrow 0$. Although it is based on integrators, (3) is noncanonical. In the literature, several similar models have also been reported[10,11] and none of them is canonical. Therefore, we propose here an extremely simple fourth-order canonical model which is controlled via the single parameter ϵ and is capable of producing chaos both with symmetrical-type and antisymmetrical-type nonlinearities. Any canonical model can be realizable directly via cascaded integrators.

This proposed model is given by

$$-\ddddot{X} = \dddot{X} + \ddot{X} + \epsilon \dot{X} + \epsilon X + f(X,\dot{X}), \tag{4}$$

where $f(X,\dot{X})$ is a nonlinear function.

We demonstrate here two cases for f. First, the antisymmetrical case where

$$f(X,\dot{X}) = f(\dot{X}) = \begin{cases} \dot{X} & \dot{X} \geqslant 1, \\ 0 & \dot{X} < 1. \end{cases} \tag{5}$$

This nonlinearity depends only on the velocity vector $\dot{X}$ and can be physically realized using a diode. The chaotic attractor observed in this case with $\epsilon = 0.1$ is plotted in Fig. 5(a) in the $X-\dot{X}$ plane.

Second, the symmetrical case where f is given by

$$f(X,\dot{X}) = f(X) = \begin{cases} -1 & X \geqslant 0, \\ 1 & X < 0. \end{cases} \tag{6}$$

The observed chaotic attractor in this case is that of Fig. 5(b). Here, $\epsilon = 0.25$ and it is clear that f depends only on the position vector X. This nonlinearity can be realized using a bipolar comparator.

Equation (4) can be rewritten in matrix form as

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \\ \dot{W} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\epsilon & -\epsilon - b & -1 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \\ W \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -a \end{pmatrix} \tag{7}$$

given that $Y = \dot{X}$, $Z = \ddot{X}$ and $W = \dddot{X}$. a and b are switching constants which reflect the type on nonlinearity in the system. In the case when f is given by (5), they are

$$a = 0 \quad \text{and} \quad b = \begin{cases} 1 & Y \geqslant 1, \\ 0 & < 1, \end{cases} \tag{8}$$

while in the case where f is given by (6), they are

$$b = 0 \quad \text{and} \quad a = \begin{cases} -1 & X \geqslant 0, \\ 1 & X < 0. \end{cases} \tag{9}$$

From (7) it is clear that when $a = 0$, the system has a single equilibrium point at the origin which is real in the region $Y < 1$ and virtual in the other region. For $\epsilon = 0.1$, the eigenvalues at this equilibrium point are $(\pm j0.333, -0.507 \pm j0.804)$ and $(-0.1, -1, 0.05 \pm j)$ in the two regions, respectively. It was found that chaos is maintained only in the range $0.005 < \epsilon < 0.2$. For $\epsilon < 0.005$, a limit cycle is sustained while for $\epsilon > 0.2$, unbounded trajectories occur. Chaotic systems with single equilibrium points have been studied in detail in Ref. 12.

In the case where f is symmetrical, i.e., $b = 0$, it is clear that the system has the two equilibrium points $(\pm 1/\epsilon, 0, 0, 0)$ both of which are real with the same set of eigenvalues $(-0.56 \pm j0.713, 0.055 \pm j0.551)$ for $\epsilon = 0.25$. The range in which the model produces chaos was found to be $0.15 < \epsilon < 0.45$. For $\epsilon < 0.15$, a limit cycle is always sustained while for $\epsilon > 0.45$, trajectories diverge and become unbounded as the two equilibrium points become too near to each other.

It is worth noting that canonical third-order models exhibiting chaos were proposed in Refs. 3, 4, 13, and 14. It is also worth noting that (4) can be chaotic in its general form $-\ddddot{X} = a\dddot{X} + b\ddot{X} + c\dot{X} + dX + f(X,\dot{X})$; this can be verified using $a = 5$, $b = 3$, $c = d = 2$ and f as in (6).

IV. CONCLUSION

We have introduced a simple integrator-based chaotic oscillator structure on the functional design level. Many circuit realizations can be derived from this structure. Of course, functional level simulation sets the boundaries for circuit-design parameters. We have also proposed a canonical fourth-order model which is capable of producing chaos using symmetrical and antisymmetrical nonlinear functions. This work is a further contribution to a long series of articles concerned with methods of designing autonomous chaotic oscillators. Another series of articles, concerned with designing nonautonomous chaotic oscillators can be found for example in Ref. 15–20. In conclusion, the area of chaotic oscillator design has now advanced to the level where it is possible to select from a wide variety of different circuits; unfortunately applications have not advanced as such.

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