All possible canonical second-order three-impedance class-A and class-B oscillators

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A report is presented on all the possible canonical second-order single-transistor three-impedance oscillators that can be derived from the general class-A or class-B two-port network topologies. Two of the ten reported cases are already known as the class-A and class-B Colpitts oscillators. A selected case, which has the appealing feature of extending the oscillation frequency beyond the resonance frequency, is chosen for experimental validation.

Introduction: Two-port network modelling offers a high level of abstraction which enables device-independent circuit structure classification [1, 2]. In a recent publication [3], single-transistor three-impedance oscillators were classified as class-A, class-B or class-C using abstract two-port network modelling. Fig. 1 shows all possible three-impedance topologies that can be connected to a two-port network. Based on which terminal is grounded, the three topologies are classified as class-A, class-B or class-C, where the terminals A, B and C are marked within Fig. 1. It was particularly shown in [3] that class-C cannot yield an oscillator whether it is implemented using a BJT or an MOS transistor while the class-A and class-B topologies have two distinct characteristic equations given respectively by

$$Z_1 + Z_2 + Z_3 - g_m Z_1 Z_3 = 0 \text{ class-A} \qquad (1a)$$

$$Z_1 + Z_2 + Z_3 + g_m Z_1 Z_2 = 0 \text{ class-B} \qquad (1b)$$

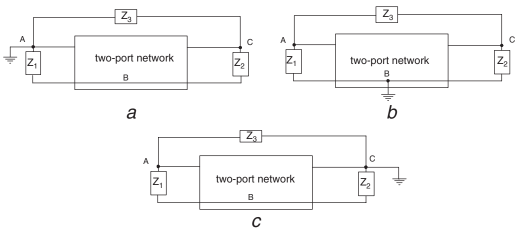

Fig. 1 *Three possible topologies for three-impedance two-port network structure*

a Class-A
b Class-B
c Class-C

where g_m is the small-signal transconductance of a BJT or an MOS transistor. More sophisticated versions of (1), which take into consideration parasitic capacitances and other non-ideal effects, where derived in [3] based on general two-port transmission parameters. However, equations (1a and b) are the starting point for oscillator circuit design by properly choosing the impedances $Z_{1,2,3}$ such that there exists a positive value for g_m to start-up oscillations and result in a positive oscillation frequency ω_o.

Here we report all the valid possibilities of Z_1, Z_2 and Z_3 which result in a canonical second-order oscillator for both class-A and class-B. A canonical second-order circuit employs a maximum of two resistors. The constraint in [3] that two capacitors and one inductor must be used, which is relevant only to the Colpitts oscillator structure, is lifted while the second-order constraint is imposed.

Table 1: All possible cases for class-A topology which yield a canonical second-order oscillator

Case	Z_1	Z_2	Z_3	g_m	ω_o^2
1	$1/sC_1$	$1/sC_2$	$r + sL$	$\frac{r}{L/C_1}$	$\frac{1}{LC_{eff}} - \frac{r^2}{L^2}$
2	$1/sC_1$	$1/sC_2$	$R\|(r + sL)$	$\frac{r}{L/C_1} + \frac{1+(C_1/C_2)}{R}$	$\frac{1}{LC_{eff}} - \frac{r^2}{L^2}$
3	$1/sC_1$	$R + 1/sC_2$	$r + sL$	$\frac{r+R}{L/C_1}$	$\frac{1}{LC_{eff}} - \frac{r(r+R)}{L^2}$
4	$1/sC_1$	R	$r + sL$	$\frac{r+R}{L/C_1}$	$\frac{1}{LC_1} - \frac{r(r+R)}{L^2}$
5	$r + sL$	$1/sC_2$	$R + 1/sC_3$	$\frac{r+R}{(L/C_3)+rR}$	$\frac{1}{LC_{eff}}\left(\frac{L+rRC_3}{L-R^2C_3}\right) - \frac{r(r+R)}{L^2-LR^2C_3}$
6	$R_1\|(1/sC_1)$	$1/sC_2$	R_3	$\frac{1}{R_3} + \frac{1+(C_1/C_2)}{R_1}$	$\frac{1}{R_1R_3C_1C_2}$

Canonical class-A oscillators: A Matlab code was written to symbolically find all valid characteristic equations of this class and eliminate valid third-order cases as well as non-canonical second-order ones. The remaining are 12 valid circuits. Noting from (1a) that Z_1 and Z_3 are interchangeable means that effectively there are only six different circuits, as summarised in Table 1. The ideal start-up value of g_m, which corresponds to a Hopf-type bifurcation and the appearance of a pure imaginary eigen-pair, is given in Table 1 alongside the oscillation frequency ω_o.

It can be seen that ω_o for the first three cases is a function of $C_{eff} = C_1C_2/(C_1 + C_2)$. However, only the first and third cases have a start-up condition independent of C_2 allowing for ω_o to be independently tuned through C_2 without affecting the start-up condition. It is also clear that cases 4 and 6 are the only two cases which employ two reactive elements and are truly second-order oscillators while the other cases employ three reactive elements and are degenerate second-order systems. Case 5 is clearly not a practical case owing to the complex expression of ω_o. In conclusion, the practically appealing circuits seem to be those of cases 1 and 3 if independent frequency tuning is required and cases 4 and 6 if only two reactive elements are to be used. Note that case 1 is already known as the class-A Colpitts oscillator [3].

Table 2: All possible cases for class-B topology which yield a canonical second-order oscillator

Case	Z_1	Z_2	Z_3	g_m	ω_o^2
1	$1/sC_1$	$1/sC_2$	$r + sL$	$\frac{r}{L/(C_1+C_2)}$	$\frac{1}{LC_{eff}}$
2	$1/sC_1$	$1/sC_2$	$R\|(r + sL)$	$\frac{(r+R)(C_1+C_2+rRC_1C_2/L)}{R^2C_{eff}-L}$; $L < R^2C_{eff}$	$\frac{(1+r/R)^2}{LC_{eff}-(L/R)^2}$
3	$1/sC_1$	$R\|(1/sC_2)$	$r + sL$	$\frac{r+R}{R^2C_2/C_1} + \frac{r}{L/(C_2+C_1+rC_1/R)}$	$\frac{1}{LC_{eff}} + \frac{r/R}{LC_2}$
4	$1/sC_1$	$R + 1/sC_2$	$r + sL$	$\frac{C_1+C_2}{[L/(r+R)]-RC_2}$; $L = mR(r+R)C_c$, $m > 1$	$\frac{m}{m-1}\frac{1}{LC_{eff}}$

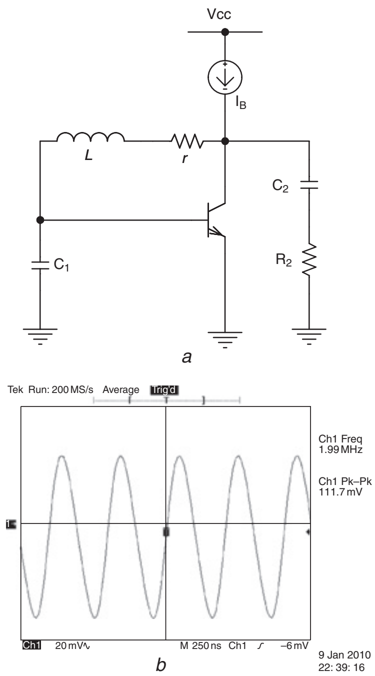

Fig. 2 *Circuit realsing case 4 of Table 2, and experimental waveform across* R_2

a Circuit realsing case 4 of Table 2
b Experimental waveform across R_2

Canonical class-B oscillators: Using a Matlab code to evaluate the different possibilities of this class and noting from (1b) that Z_1 and Z_2 can be symmetrically interchanged, only four canonical second-order circuits where found, as listed in Table 2. Case 1 is already known as the class-B Colpitts oscillator [3]. Note from Table 2 that case 2 and case 4 impose a constraint on the value of L. However, case 3 and case 4 have an appealing feature where ω_o can be made higher than the resonance frequency $1/\sqrt{LC_{eff}}$. This is not possible for any of the class-A cases in Table 1. In conclusion, the two most practical cases of this class are cases 1 and 4.

Experimental validation: The circuit shown in Fig. 2*a* was constructed to validate case 4 in Table 2 as an example. The components chosen were: $L = 116\ \mu H$; $r = 140\ \Omega$ ($107\ \Omega$ physical resistance + $33\ \Omega$ internal inductor resistance), $R_2 = 1\ k\Omega$; $C_1 = C_2 = 100$ pF and a Q2N2222 NPN transistor was used. These values yield $m = 2.035$ and a resonance frequency $f_{res} = 1/2\pi\sqrt{LC_{eff}} = 1.477$ MHz. Consulting Table 2, it is expected that the measured oscillation frequency should be equal to $\sqrt{m/(m-1)}f_{res} \simeq 2.07$ MHz. Fig. 2*b* shows the observed experimental waveform across R_2 with a measured oscillation frequency of 1.99 MHz very close to the expected value. The bias current to start-up oscillations was $I_B = 53\ \mu A$. Note that taking the output across R_2 has the advantage of being free from the DC biasing voltage of the employed transistor.

Conclusion: Out of 95 possible class-A oscillators and 19 possible class-B oscillators, only six class-A and four class-B were found to yield a canonical second-order oscillator and are reported here. Most of the found circuits are second-order degenerate structures which, with non-linear modelling, will remain third-order circuits [4]. However, the characteristic equations (1a and b) are valid only for small-signal linearised transistor models. As such, the second-order classification remains valid.

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