Experimental Technique For Estimating the Dispersion Coefficient of a Constant Phase Element

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Abstract– **We report an experimental technique for estimating the dispersion coefficient α of a constant phase element (CPE). The method employs using a setup to excite the impedance under consideration via a periodic triangle-wave current $I(t)$. Measuring the 50% rise time of the developed voltage $V(t)$ then enables finding α using a pre-calculated Table. Experimental results are shown.**

I. Introduction

The Cole-Cole impedance model [1], [2] is a popular method for characterizing the electrochemical properties of biological tissues and biochemical materials. The model comprises three hypothetical circuit elements: a low frequency resistor R_0, a high frequency resistor R_∞ and a Constant Phase Element (CPE), arranged as shown within Fig. 1. The CPE is also known as the fractional capacitor [3] and its impedance is $Z_{CPE} = 1/(j\omega C)^\alpha$ where C is the capacitance and α is its order $(0 < \alpha \leq 1)$. The Cole-Cole impedance is given by

$$Z = R_\infty + \frac{R_0 - R_\infty}{1 + (j\omega\tau)^\alpha} = Z' + jZ'' \qquad (1)$$

and $(j\omega)^\alpha = \omega^\alpha[\cos(\alpha\pi/2) + j\sin(\alpha\pi/2)]$.

It is clear that the heart of this model is the Constant Phase Element (CPE) or fractional capacitor [3] who's order α is also known as the dispersion coefficient $(0 < \alpha \leq 1)$. Finding the value of α is of particular importance for specific applications; mainly as a medical diagnostic tool [4]-[6]. In some applications, using a CPE, rather than a complete Cole-Cole model (which includes two additional resistors, see Fig. 1), is sufficient enough [7].

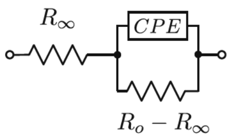

Figure 1: The Cole-Cole impedance model

The traditional way of finding α implies using an impedance analyzer to plot the real part of the measured impedance (Z') versus the imaginary part (Z''), as shown in Fig. 2. A semi-circular arc is obtained (usually using least squares regression) and measuring the angle $\varphi_{CPE} = \alpha\pi/2$ enables calculating α (see Fig. 2). An alternative method, which does not need an expensive impedance analyzer, was recently reported in [8]. However, finding α requires solving numerically the following equation [8]

$$\left(\frac{G_1 - 2G_2}{\sqrt{G_1 G_2}}\right)\left(\cos(\frac{\alpha\pi}{2}) + \sqrt{\frac{b + \cos(\alpha\pi)}{2}}\right) = p^\alpha \qquad (2)$$

where G_1, G_2, b and p are constants that need to be measured before solving (2). This requires a microprocessor, as shown in Fig. 3, which represents a device designed based on the technique in [8].

In this work, we report an alternative experimentally-oriented method that requires no calculations to estimate α. We verify the proposed method on a number of fruit and vegetable tissues [7]. We stress that this method enables finding only α but not the complete Cole-Cole model.

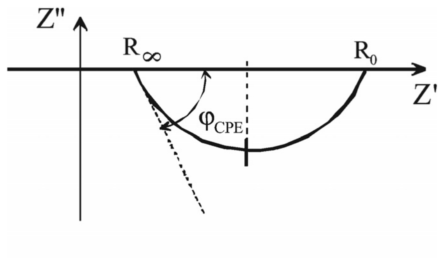

Figure 2: Impedance loci used to extract α.

II. Proposed Technique

A. Mathematical foundation

In [9], the following expression was given for the voltage developed across a Constant Phase Element when excited by a periodic (with period T) triangle-wave current $I(t)$

$$V(t) = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^{2-\alpha}} \cdot \left[\sin_\alpha\left((2n-1)t\right)\cos(\frac{\alpha\pi}{2}) + \cos_\alpha\left((2n-1)t\right)\sin(\frac{\alpha\pi}{2})\right] \quad (3)$$

where $\sin_\alpha(\omega t)$ and $\cos_\alpha(\omega t)$ are the generalized trigonometric functions of order α, defined respectively as [10]

$$\sin_\alpha(t) = \sum_{k=0}^{\infty} e_{k-\alpha}^{t} \sin(k-\alpha)\frac{\pi}{2} \quad (4a)$$

$$\cos_\alpha(t) = \sum_{k=0}^{\infty} e_{k-\alpha}^{t} \cos(k-\alpha)\frac{\pi}{2} \quad (4b)$$

Accordingly, it is possible to show that the steady-state voltage will be given by

$$V_{ss}(t) = \lim_{t\to\infty} V(t) = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^{2-\alpha}} \sin\left[(2n-1)t + \frac{\alpha\pi}{2}\right] \quad (5)$$

which is also periodic with period T.

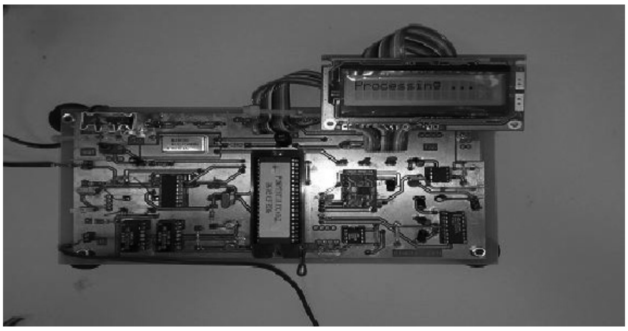

Figure 3: Cole-Cole meter device developed based on the technique proposed in [8]

α	t_r/T	α	t_r/T
0.5	9.42%	0.7	3.26%
0.6	6.25%	0.8	1.06%
0.9	0.11%		

Table 1: Measured t_r/T for different α

Figure 4 is a plot of $V_{ss}(t)$ for values of $\alpha \in [0.5, 0.9]$ in steps of 0.1. The zero crossing point of each response corresponds to the 50% rise-time t_r. Therefore, by measuring the ratio t_r/T, one can easily estimate α after consulting Fig. 4. A look-up table for α with any required precision can be constructed using (5). Table 1 shows the measured (t_r/T) for some different values of α. In the range $\alpha \in [0.5, 0.95]$, we also found the best fit polynomial to be $(t_r/T)\% = -1432\alpha^5 + 5172.7\alpha^4 - 7299.7\alpha^3 + 5063.6\alpha^2 - 1764.3\alpha + 259.64$.

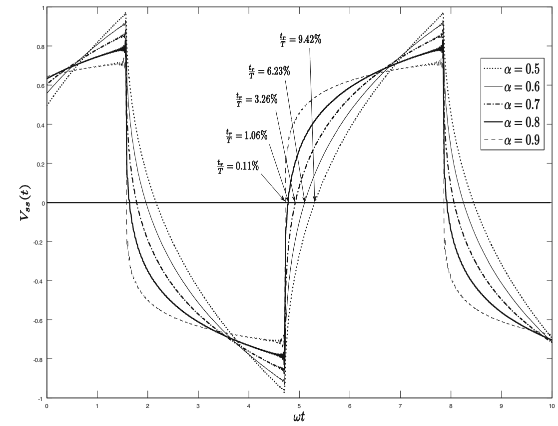

Figure 4: Waveforms of the steady-state voltage obtained by numerically solving (5).

B. Experimental results

Figure 5 shows the experimental setup used to measure t_r. Here, two AD844 current feedback op amps and one resistor R_v are used. Op amp U_1 converts the applied exciting triangle waveform from voltage to current which is then buffered by op amp U_2 and applied to the impedance under consideration. The simplicity of the circuit is evident when compared to the system in Fig. 3.

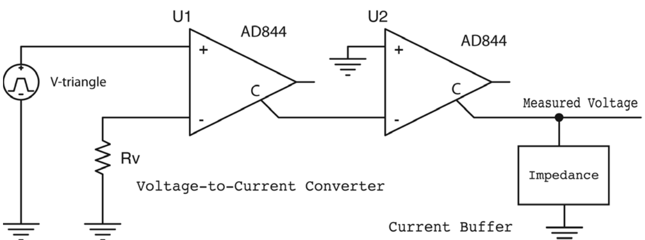

Figure 5: Experimental setup used to excite the impedance and measure the rise-time.

Figure 6(a) shows the measurements for three different fruit samples. The corresponding t_r/T ratios are approximately 10% for the Apple, 3.8% for the Orange and 1.8% for the Tomato and hence the values of α are estimated to be 0.5, 0.675 and 0.753 respectively. Figures 6(b) and 6(c) show the measurement from the tomato fruit and the Apple fruit compared to the case when they would be replaced with normal dielectric capacitors of the same capacitance values. Of course, with a dielectric capacitor, the developed voltage $V(t)$ is very close to an ideal square-wave. The excitating triangle waveform is also shown within the figures.

III. Conclusion

We have described an experimental technique to estimate the dispersion coefficient α based on finding the zero-crossing rise-time of the voltage developed across the impedance under consideration as a result of a triangle-wave current exciting signal.

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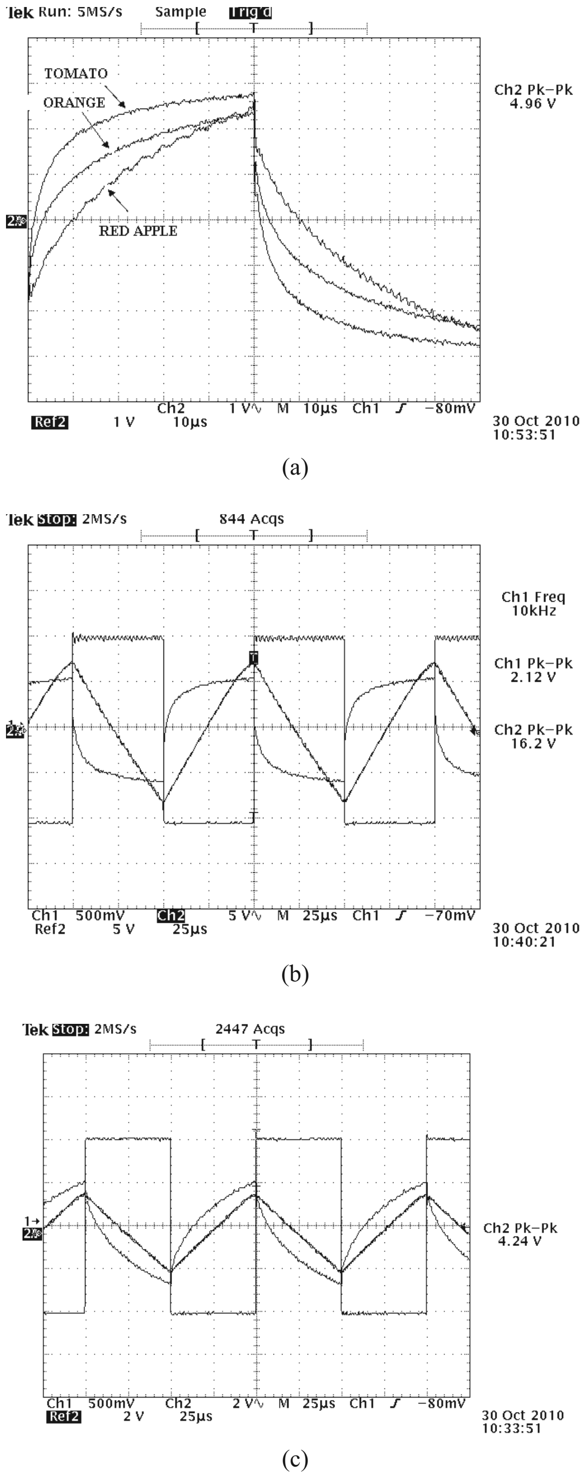

Figure 6: Measurements of the rise-time using the circuit in Fig. 5; (a) $V(t)$ across three different fruits; (b)$V(t)$ across a tomato fruit compared to the excitation and the ideal response of a dielectric capacitor and (c) $V(t)$ across an apple fruit.