Extracting the Cole-Cole impedance model parameters without direct impedance measurement

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A filter-based technique to extract the four parameters that characterise a single-dispersion Cole-Cole impedance model without direct measurement of the impedance real and imaginary parts is reported. Experimental results in the range 100 Hz—5 MHz using apples, apricots and kiwi fruits are given and results are compared with numerical simulations of the acquired model.

Introduction: The Cole-Cole impedance model [1, 2] is an elegant and widely popular method for characterising the electrochemical properties of biological tissues and biochemical materials [3]. The model comprises three hypothetical circuit elements: a low-frequency resistor R_0, a high-frequency resistor R_∞ and a constant phase element (CPE), arranged as shown in Fig. 1. The CPE is also known as the fractional capacitor [4], and its impedance is $Z_{CPE} = 1/(j\omega C)^\alpha$ where C is the capacitance and α is its order ($0 < \alpha \leq 1$). The Cole-Cole impedance is given by

$$Z = R_\infty + \frac{R_0 - R_\infty}{1 + (j\omega\tau)^\alpha} = Z' + jZ'' \tag{1}$$

and $(j\omega)^\alpha = \omega^\alpha[\cos(\alpha\pi/2) + j\sin(\alpha\pi/2)]$.

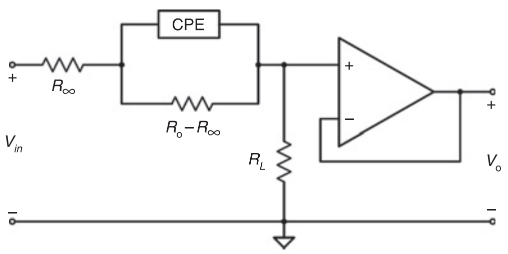

Fig. 1 *Proposed filter structure with Cole-Cole impedance terminated by load resistance R_L*

To characterise a particular tissue [5], finding the values of the four parameters (R_0, R_∞, τ, α) is required. Note that τ is known as the characteristic time constant of the tissue and is equal to $[(R_0 - R_\infty)C]^{1/\alpha}$. To extract these values, an impedance analyser is used to measure the impedance of the tissue under consideration and a plot is then constructed, as shown in Fig. 2, relating the imaginary part Z'' to the real part Z'. A circular arc is obtained (usually using least squares regression) from which R_0 and R_∞ can be directly found and the angle $\varphi_{CPE} = \alpha\pi/2$ enables calculating α (see Fig. 2). Finally, the frequency at which $|Z'|$ has its maximum value is equal to $1/\tau$. This technique has been employed in all tissue characterisation applications using the Cole-Cole model [6–8] from which it is clear that acquiring Z' and Z'' requires the use of an expensive impedance analyser and post processing of the data. Cheap alternatives include using data acquisition cards and custom software modules [9].

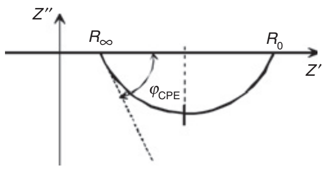

Fig. 2 *Impedance loci used to extract parameter set (R_0, R_∞, τ, α)*

In this Letter we introduce a filter-based measurement technique which enables extracting the values of the parameter set (R_0, R_∞ τ, α) without having to measure the impedance. We apply the developed theory to apples, apricots, kiwis and potatoes to extract their tissue properties. The obtained results are consistent, yet more accurate than those reported in [10].

Filter-based setup: Fig. 1 shows the simple filter setup where a load resistance R_L terminates the Cole impedance. An opamp buffer with suitable bandwidth is used to measure the output voltage V_o. It can be shown that the transfer function of this filter is

$$T(s) = \frac{V_o}{V_i} = \frac{1}{G_1} \times \frac{1 + (\tau s)^\alpha}{1 + \frac{G_2}{G_1}(\tau s)^\alpha} \tag{2}$$

where $G_1 = 1 + (R_0/R_L)$, $G_2 = 1 + (R_\infty/R_L)$ and $s = j\omega$. It is clear that $G_2/G_1 < 1$ and hence the magnitude response $|T(j\omega)|$ will always show a highpass response. We then note the following:

(i) at $\omega = 0 \rightarrow |T(j\omega)| = 1/G_1$. Hence, measuring the DC gain and knowing R_L yields R_0.
(ii) at $\omega = \infty \rightarrow |T(j\omega)| = 1/G_2$. Hence, measuring the high frequency gain and knowing R_L yields R_∞.
(iii) the 3dB point at which $|T(j\omega_{3dB})| = 1/\sqrt{2}G_2$ is given by

$$\omega_{3dB} = \frac{a}{\tau}\left(\cos\left(\frac{\alpha\pi}{2}\right) + \sqrt{\frac{b + \cos(\alpha\pi)}{2}}\right)^{1/\alpha} = \frac{f_1(\alpha)}{\tau} \tag{3a}$$

$$\text{with } a = \left(\frac{G_1}{G_2} - 2\right)^{1/\alpha} \text{ and } b = \frac{G_1(3G_1 - 4G_2)}{(G_1 - 2G_2)^2} \tag{3b}$$

(iv) the phase angle $\angle T(j\omega)$ exhibits a maximum value at a frequency $\omega_{\phi\max}$ given by

$$\omega_{\phi\max} = \frac{1}{\tau}\left(\sqrt{\frac{G_1}{G_2}}\right)^{1/\alpha} = \frac{f_2(\alpha)}{\tau} \tag{4}$$

and at this frequency, the magnitude $|T(j\omega_{\phi\max})|$ is uniquely equal to $1/\sqrt{G_1 G_2}$. Therefore, without plotting the phase, it is possible to find $\omega_{\phi\max}$ from the magnitude response.
(v) from (3) and (4), it is seen that the ratio $\omega_{3dB}/\omega_{\phi\max} = f_1(\alpha)/f_2(\alpha)$ is independent of τ and a function only of α. Therefore assuming $\omega_{3dB}/\omega_{\phi\max} = p$, α is found by numerically solving the equation

$$\left(\frac{G_1 - 2G_2}{\sqrt{G_1 G_2}}\right)\left(\cos\left(\frac{\alpha\pi}{2}\right) + \sqrt{\frac{b + \cos(\alpha\pi)}{2}}\right) = p^\alpha \tag{5}$$

Knowing α, τ could then be found from (4).

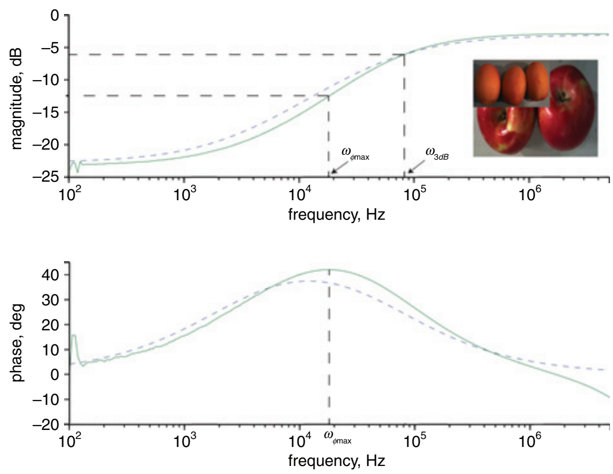

Fig. 3 *Experimental (solid) and Matlab simulation (dashed) plots of magnitude and phase response of apricot fruit using filter structure of Fig. 1*

Experimental validation: Fig. 2 was constructed using an OP27 opamp and $R_L = 1$ kΩ to extract the Cole model parameters for an apricot fruit in the range 100 Hz—5 MHz. Fig. 3 shows the observed magnitude response from which the DC gain was measured as −22.857 dB which yields $R_0 = 12.894$ kΩ and the high frequency gain was measured as −2.958 dB yielding $R_\infty = 405.7$ Ω. The frequency ω_{3dB} at which the magnitude is −5.875 was located at 85.92 kHz while $\omega_{\phi\max}$ was located at 19.069 kHz corresponding to a magnitude of −12.907 dB. Thus $\omega_{3dB}/\omega_{\phi\max} = 4.505$ and numerically solving (5) yields $\alpha = 0.739$ and hence $\tau = 62.34$ μs. In Fig. 3, we have also plotted the phase response of the filter in order to show the existence of a maximum phase, as theoretically predicted. However, the phase response is not needed to extract the model parameters as explained above. The Matlab magnitude and phase response simulations using the extracted parameters are also plotted in Fig. 3 (dashed lines) showing very good agreement with the experimental measurements within the limits of accuracy of the Cole model itself [2]. Measurements were repeated for two more apples, two kiwis and two potatoes. Results are summarised in Table 1 where the extracted capacitance C is also given. We note that the impedance-based method used in [10] to obtain the same fruit parameters is not only more difficult but is also less accurate as it relies on linearised segment slopes in the frequency domain.

Table 1: Experimentally extracted Cole model parameters (resistance in kΩ)

	R_0	R_∞	α	τ (μ sec)	C (nF)
Apricot 1	12.894	0.406	0.739	62.34	63
Apricot 2	12.902	0.379	0.751	69.69	60
Apple 1	29.637	2.72	0.633	120.77	122
Apple 2	22.493	2.511	0.678	96.8	88
Kiwi 1	5.025	0.26	0.669	46.59	265
Kiwi 2	5.939	0.32	0.651	52.78	292
Potato 1	7.128	0.323	0.731	45.12	98
Potato 2	6.306	0.25	0.741	36	84

Conclusion: In a filter setup that incorporates a tissue and a load resistance, and by plotting only the magnitude response, all parameters of the Cole-Cole model that characterise this tissue can be extracted without need for an impedance analyser. The only computational overhead is the need to numerically solve (5) for α. To the best of the authors' knowledge, this is the simplest possible method yet reported.

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One or more of the Figures in this Letter are available in colour online.

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