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Design of non-balanced cross-coupled oscillators with no matching requirements

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Abstract: Using two-port network transmission parameters we derive the general characteristic equation of a cross-coupled circuit topology which involves two active devices and four or six impedances. The derived equations are generic and apply both to BJT or MOS transistors and even any other active device without any matching constraints. Application to realising novel non-balanced non-matched cross-coupled oscillators is demonstrated. Spice simulations of a MOS oscillator in a 0.25 μ technology are given as well as experimental results from an oscillator employing discrete Bipolar transistors.

1 Introduction

Cross-coupled oscillators are very important and widely used in the realisation of transceivers. A huge amount of literature pertaining to the design and analysis of these oscillators both in their sinusoidal oscillation mode and their relaxation oscillation mode can be found [1–7]. Two typical widely used cross-coupled oscillator structures are shown in Figs. 1*a* and *b* employing MOS transistors. The same structures may well be implemented using Bipolar transistors. In both structures, the design requirements usually imposed are:

1. the two transistor should be matched in order to realise an effective differential negative resistance $r = -1/g_m$ where g_m is the small signal transconductance for each individual transistor;

2. the load impedances of both transistors should be similar in order to obtain a symmetrical (balanced) structure.

It is not however clear whether these design requirements are essential or not in order to obtain an oscillator.

In this work, we derive using two port network transmission parameters the characteristic equation of the general four-impedance cross-coupled structure shown in Fig. 1*c* and the six-impedance structure shown in Fig. 1*d* without assuming that the employed transistors have to be matched. The derived equation is valid both for BJT and MOS transistors and indeed for any other active device. We then find the oscillation start-up condition and oscillation frequency for the two classical cases shown in Fig. 1 and for other cases in which the two load impedances (Z_2 and Z_3) are not similar; that is the cross-coupled oscillator in this case is not balanced. Of course, oscillators whether sinusoidal or relaxation are highly non-linear circuits and their accurate modelling and analysis implies using non-linear dynamics techniques [8–10]. However, linearised small-signal models are usually the start point for the design phase of any oscillator in order to derive the oscillation start-up condition (Hopf bifurcation condition) and also estimate the expected oscillation frequency as a function of the circuit parameters. By virtue of using transmission parameters, which are particularly suited for networks in cascade, we show how the effects of parasitic impedances can be easily incorporated into the derived characteristic equation. Two novel design examples of cross-coupled oscillators where (i) the transistors are not matched and are even of non-identical types and (ii) the loads are not balanced are given. Spice simulation using a 0.25 μ MOS technology file is used to verify one example while the other is verified experimentally using discrete components.

2 Background

Fig. 2*a* shows the typical two-port network input and output variables for which the transmission matrix (***a***) is

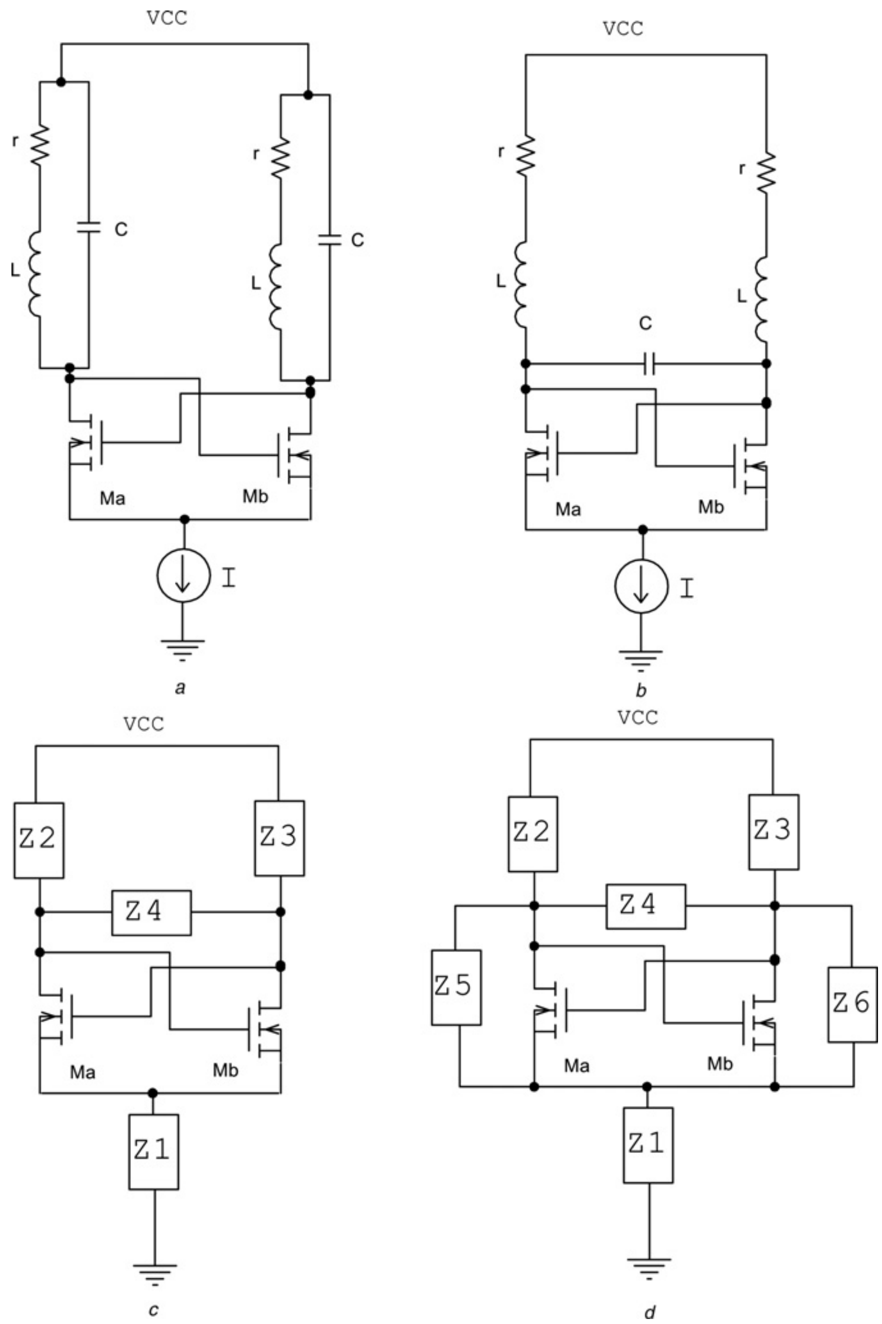

Figure 1 *Cross-coupled oscillator structures*

a and *b* Two classical balanced-load cross-coupled oscillators
c Generic structure with four impedances
d Generic structure with six impedances

defined as

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} \tag{1}$$

where V_1 and I_1 are the input port voltage and current, while V_2 and I_2 are the output port voltage and current, respectively. Fig. 2*b* shows a basic small signal equivalent circuit of an NPN BJT transistor operating in the forward active mode. The model comprises two terminal resistors (r_b, r_e) (ideally $r_e = 0$) and a dependent current-controlled current source βI_1 with output resistance r_o. β is the forward active current gain (assumed constant) and r_b is related to the BJT small-signal transconductance g_m as $r_b = \beta/g_m$. It can be shown that the (***a***) matrix for this equivalent circuit is [11]

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \frac{-1}{\beta r_o - r_e} \begin{pmatrix} r_b + r_e & \beta r_e r_o + r_b(r_o + r_e) \\ 1 & r_o + r_e \end{pmatrix} \tag{2}$$

The above (*a*) may be simplified considering that $\beta r_o \gg r_e$

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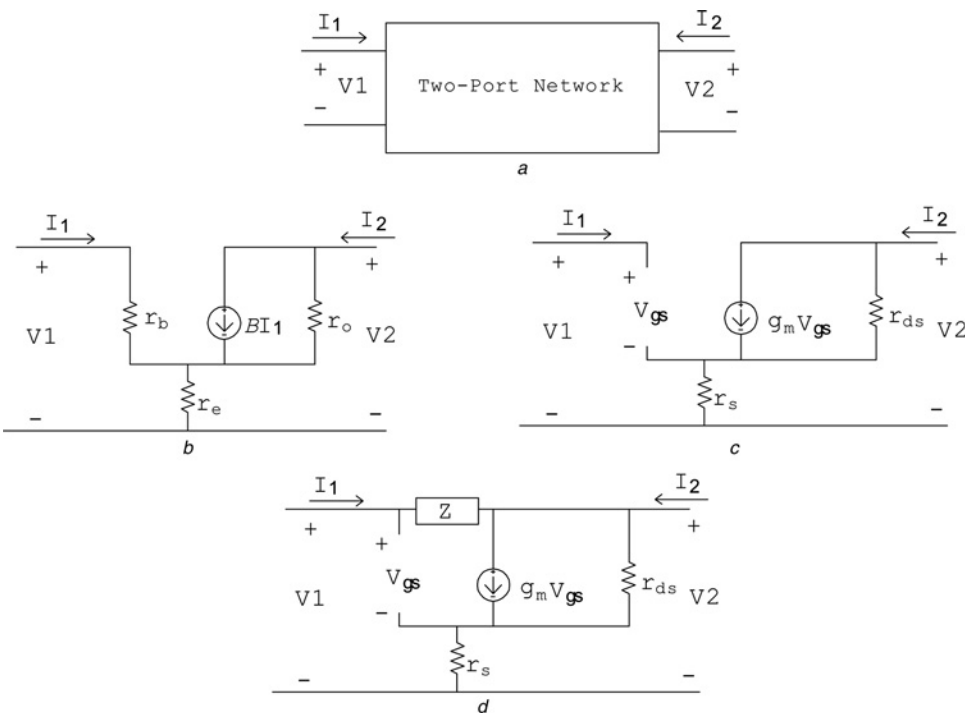

Figure 2 *Two-port network representations*

a Two-port network variables
b Basic small signal equivalent model of a BJT transistor
c Basic small signal equivalent model of a MOS transistor
d MOS transistor model with a gate-to-drain impedance *Z*

and $(1/\beta r_o) \to 0$ and then becomes

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} \frac{-1}{A} & -\left[r_e\left(1+\frac{1}{A}\right)+\frac{1}{g_m}\right] \\ 0 & \frac{-1}{\beta} \end{pmatrix} \quad (3)$$

where $A = g_m r_o$. If A is sufficiently large such that $(1/A) \to 0$, a further simplification yields

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 0 & -\left(r_e+\frac{1}{g_m}\right) \\ 0 & \frac{-1}{\beta} \end{pmatrix} \quad (4)$$

In a similar manner, and recalling the small signal equivalent circuit of a MOS transistor operating in the saturation mode (Fig. 2*c*), it can be shown that the (***a***) matrix for this circuit is

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \frac{-1}{g_m r_{ds}} \begin{pmatrix} 1 & r_s(1+g_m r_{ds})+r_{ds} \\ 0 & 0 \end{pmatrix} \simeq \begin{pmatrix} \frac{-1}{g_m r_{ds}} & -\left(r_s+\frac{1}{g_m}\right) \\ 0 & 0 \end{pmatrix} \quad (5)$$

where g_m is the small signal transconductance, r_{ds} is the drain to source resistance and r_s is the source resistance (ideally $r_s = 0$). It is worth noting that under the ideal conditions $(r_e, r_o, \beta) = (0, \infty, \infty)$ for the BJT and $(r_s, r_{ds}) = (0, \infty)$ for the MOS, both transistors can be described by the ideal transmission matrix

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} 0 & -1/g_m \\ 0 & 0 \end{pmatrix} \quad (6)$$

where the transconductance g_m is the only design parameter.

However, more complex transistor models can also be accommodated for. Consider for example the MOS transistor small signal model of Fig. 2*d* where an impedance Z (typically a parasitic C_{gd} capacitor) is connected from gate to drain. The transmission matrix in this case can be shown to be

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \frac{1}{r_{ds}(1-g_m Z)+r_s} \times \begin{pmatrix} r_{ds}(1+g_m r_s)+Z & r_{ds}(1+g_m r_s)Z \\ 1+g_m r_{ds} & r_{ds}(1+g_m r_s) \end{pmatrix} \quad (7)$$

Further, and since transmission parameters are well suited for networks in cascade, we may additionally incorporate the effects of C_{gs} and C_{ds} (treating each as an individual two-port network) via matrix multiplication with (7) in the order of the cascade yielding

$$\begin{pmatrix} 1 & 0 \\ sC_{gs} & 1 \end{pmatrix} \cdot \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ sC_{ds} & 1 \end{pmatrix} = \begin{pmatrix} a_{11} + sC_{ds}a_{12} & a_{12} \\ a_{21} + sC_{gs}(a_{11} + sC_{ds}a_{12}) + sC_{ds}a_{22} & a_{22} + sC_{gs}a_{12} \end{pmatrix} \quad (8)$$

It is thus clear that the complexity of the transistor model automatically reflects in the elements of the matrix. As such, a two-port network analysis of the cross-coupled structure of Fig. 1*c* would render a general characteristic equation which

1. is independent of any particular active device model;

2. can automatically absorb any level of complexity in the active device model without needing to re-analyse the oscillator to derive a new characteristic equation every time the model is changed;

3. can be used to study the possibility of having non-matched and non-balanced oscillators; which is the target of this work.

It is also worth noting that the (***a***) matrix elements can be experimentally found via a network analyser.

3 Four-impedance cross-coupled structure

Consider the two-port network representation of Fig. 1*c*, shown in Fig. 3*a*. Assuming the two transistors are not similar (This is more general than assuming they are not matched.) implies they are, respectively, represented by the distinct transmission matrices (***a***) and (***b***); as defined by (1). Noting from Fig. 3*a* that $V_1 = V_4$, $V_2 = V_3$ and $I_1 + I_2 + I_3 + I_4 = 0$, it can then be shown that a characteristic equation of Fig. 3*a* exists only if $Z_1 \to \infty$ and is given by (Z_1 is therefore usually replaced by a DC biasing current source.)

$$\frac{1 + Z_3\left(\frac{1}{Z_4} + \frac{b_{11}}{b_{12}} + \frac{a_{22}}{a_{12}}\right) + Z_2\left(\frac{1}{Z_4} + \frac{1}{a_{12}} + \frac{b_{11}b_{22}}{b_{12}} - b_{21}\right)}{1 + Z_3\left(\frac{1}{Z_4} + \frac{1}{b_{12}} + \frac{a_{11}a_{22}}{a_{12}} - a_{21}\right) + Z_2\left(\frac{1}{Z_4} + \frac{b_{22}}{b_{12}} + \frac{a_{11}}{a_{12}}\right)} = -\frac{b_{21} + \frac{b_{11}(1 - b_{22})}{b_{12}} - \frac{1 - a_{22}}{a_{12}}}{a_{21} + \frac{a_{11}(1 - a_{22})}{a_{12}} - \frac{1 - b_{22}}{b_{12}}} \quad (9)$$

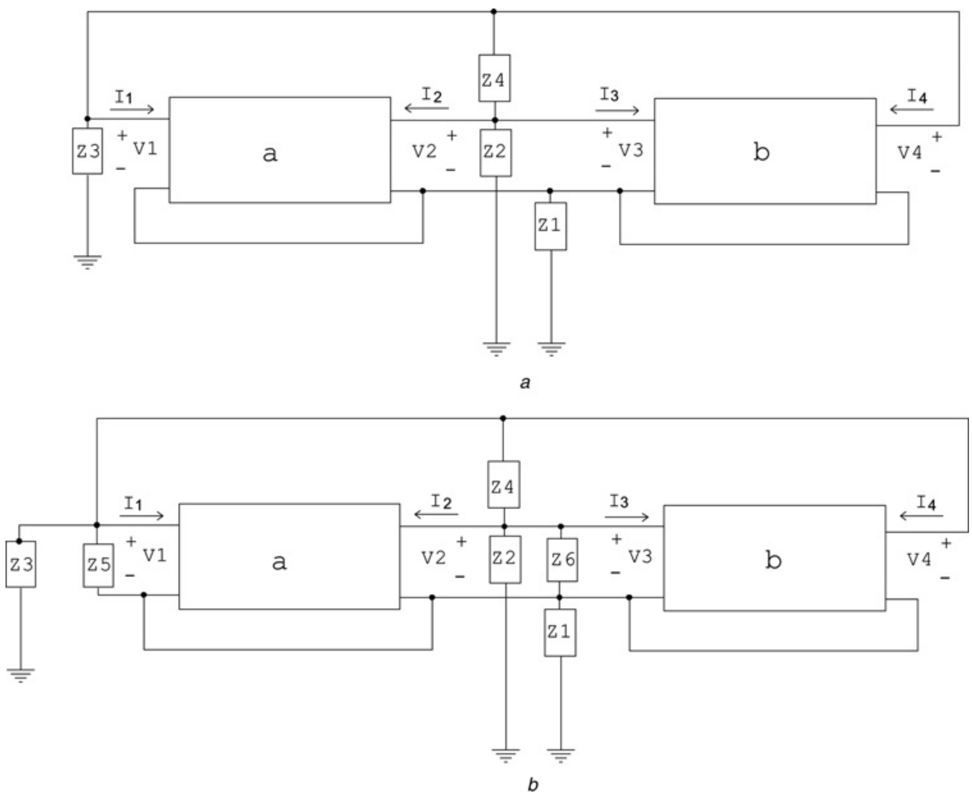

Figure 3 *Two-port network models of*

a Four-impedance cross-coupled oscillator structure
b Six-impedance structure

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If the transistors are matched and identically biased; that is $(\boldsymbol{a}) = (\boldsymbol{b})$ the above equation reduces to

$$1 + (Z_2 + Z_3)\left(\frac{1}{Z_4} + \frac{1 + a_{11} + a_{22} + a_{11}a_{22}}{2a_{12}} - \frac{a_{21}}{2}\right) = 0 \tag{10}$$

Without loss of generality, we consider the transmission matrix of (6) which applies both for the BJT and MOS transistors under ideal conditions. This does not undermine the generality of our analysis since we may instead use the more complex matrices of (2), (5), (7) or (8).

Substituting with (6) in (9) yields

$$1 + \frac{Z_2 + Z_3}{Z_4} - \frac{g_{ma}g_{mb}}{g_{ma} + g_{mb}}(Z_2 + Z_3) = 0 \tag{11}$$

where g_{ma} and g_{mb} are the transconductances of the two transistors. Interestingly, the characteristic equation (11) is dependent on the effective transconductance of the two transistors $g_{meff} = g_{ma}g_{mb}/(g_{ma} + g_{mb})$. Now, if the two transistors are matched and have balanced DC biasing then $g_{ma} = g_{mb} = g_m$ and (11) simplifies to

$$1 + (Z_2 + Z_3)\left(\frac{1}{Z_4} - \frac{g_m}{2}\right) = 0 \tag{12}$$

Obviously the choice of $g_{ma} = g_{mb} = g_m$ may simplify the design but it is not a necessary choice; that is there is no reason why the two transistors have to be strictly matched.

Reverting back to (11) four different cases of cross-coupled oscillators can be designed. In particular:

Case A: matched devices ($g_{ma} = g_{mb} = g_m$) with balanced loads ($Z_2 = Z_3$).

Case B: matched devices with non-balanced loads ($Z_2 \neq Z_3$).

Case C: non-matched devices ($g_{ma} \neq g_{mb}$) with balanced loads.

Case D: non-matched devices with non-balanced loads.

In what follows we give possible design examples of each case. However, before doing so it is also worth commenting on the six-impedance cross-coupled structure shown in Fig. 1*d*. Here, two more impedances Z_5 and Z_6 are added to complete all possible inter-node connections of the two-port representation of Fig. 3*b*. Because of using transmission parameters, one does not need to re-analyse this six-impedance structure. In particular, the characteristic equation (9), derived for the four-impedance case, will remain valid taking into consideration that the $(\boldsymbol{a})$ and $(\boldsymbol{b})$ matrices have to be modified as

$$(\boldsymbol{a})_{\text{new}} = \begin{pmatrix} 1 & 0 \\ Z_5 & 1 \end{pmatrix} \cdot (\boldsymbol{a})_{\text{old}} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} + a_{11}Z_5 & a_{22} + a_{12}Z_5 \end{pmatrix} \tag{13}$$

$$(\boldsymbol{b})_{\text{new}} = \begin{pmatrix} 1 & 0 \\ Z_6 & 1 \end{pmatrix} \cdot (\boldsymbol{b})_{\text{old}} = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} + b_{11}Z_6 & b_{22} + b_{12}Z_6 \end{pmatrix} \tag{14}$$

4 Design Examples

4.1 Cross-coupled oscillators with balanced loads

Fig. 1*a* and *b*, which are conventional cross-coupled oscillators, represent examples of balanced load oscillators which may either belong to case A or case C above. Fig. 1*a* has $Z_2 = Z_3 = Z = (sL + r)/(1 + rCs + LCs^2)$ and $Z_4 = \infty$. Hence using (11) one obtains the characteristic equation

$$1 - 2g_{meff}Z = 0 \tag{15}$$

or equivalently

$$s^2 + \left(\frac{r}{L} - 2\frac{g_{meff}}{C}\right)s + \frac{1}{LC}(1 - 2rg_{meff}) = 0 \tag{16}$$

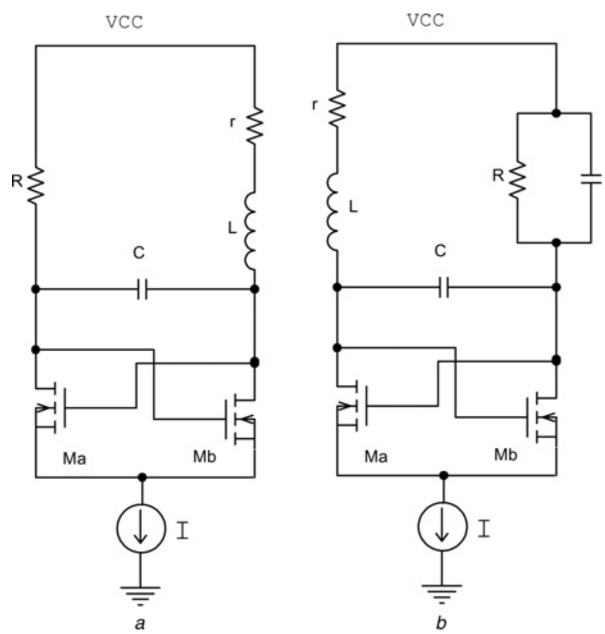

Figure 4 *Examples of non-balanced cross-coupled oscillators*

a Second-order oscillator
b Third-order oscillator

The oscillation start-up condition and oscillation frequency are thus given by

$$g_{meff} = \frac{1}{2}\frac{rC}{L} \quad \text{and}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{1-\left(\frac{r^2C}{L}\right)} = \frac{1}{\sqrt{LC}}\Big|_{r\to 0} \qquad (17)$$

In the special case of two matched and symmetrically biased transistors ($g_{ma} = g_{mb} = g_m$), the above start-up condition simply reduces to $g_m = rC/L$ maintaining the same oscillation frequency; as widely known.

For Fig. 1*b* we have $Z_2 = Z_3 = Z = sL + r$ and $Z_4 = 1/sC$. Using (11) one obtains the characteristic equation

$$1 + 2Z\left(\frac{1}{Z_4} - g_{meff}\right) = 0 \qquad (18)$$

or equivalently

$$s^2 + \left(\frac{r}{L} - \frac{g_{meff}}{C}\right)s + \frac{1}{LC}\left(\frac{1}{2} - rg_{meff}\right) = 0 \qquad (19)$$

The oscillation start-up condition and oscillation frequency are thus

$$g_{meff} = \frac{rC}{L} \quad \text{and}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}\sqrt{\frac{1}{2}-\left(\frac{r^2C}{L}\right)} = \frac{1}{\sqrt{2LC}}\Big|_{r\to 0} \qquad (20)$$

Again, if the two transistors are matched and symmetrically biased the above start-up condition reduces to $g_m = 2rC/L$ maintaining the same oscillation frequency.

Table 1 All possible non-balanced second-order cross-coupled oscillators ($R_{eff} = rR/(r + R)$, $C_{eff} = C_2C_3/(C_2 + C_3)$ and $R_t = r + R$)

Case	Z_2	Z_3	Z_4	g_{meff}	ω_0	Remarks
1	$1/sC_2$	$1/sC_3$	$r + sL$	$\frac{rC_{eff}}{L}$	$\frac{1}{\sqrt{LC_{eff}}}\sqrt{1-\frac{r^2C_{eff}}{L}}$	
2	$1/sC_2$	$1/sC_3$	$R//(r + sL)$	$\frac{rC_{eff}}{L}+\frac{1}{R}$	$\frac{1}{\sqrt{LC_{eff}}}\sqrt{1-\left(\frac{r^2C_{eff}}{L}\right)}$	
3	$1/sC$	$r + sL$	R	$\frac{1}{R_{eff}}$	$\frac{1}{\sqrt{LC}}$	
4	$1/sC_2$	$R + 1/sC_3$	$r + sL$	$\frac{1}{R_{eff}+\frac{L}{R_tC_{eff}}}$	$\frac{1}{\sqrt{LC_{eff}}}\sqrt{\frac{r^2-L/C_{eff}}{R^2-L/C_{eff}}}$	
5	$1/sC$	R	$r + sL$	$\frac{1}{R_{eff}+\frac{L}{R_tC}}$	$\frac{1}{\sqrt{LC}}\sqrt{\frac{r^2-L/C}{R^2-L/C}}$	
6	$r + sL$	$1/sC$	R	$\frac{1}{R_{eff}}$	$\frac{1}{\sqrt{LC}}$	mirror of case 3
7	$R + 1/sC_2$	$1/sC_3$	$r + sL$	$\frac{1}{R_{eff}+\frac{L}{R_tC_{eff}}}$	$\frac{1}{\sqrt{LC_{eff}}}\sqrt{\frac{r^2-L/C_{eff}}{R^2-L/C_{eff}}}$	mirror of case 4
8	R	$1/sC$	$r + sL$	$\frac{1}{R_{eff}+\frac{L}{R_tC}}$	$\frac{1}{\sqrt{LC}}\sqrt{\frac{r^2-L/C}{R^2-L/C}}$	mirror of case 5
9	R	$r + sL$	$1/sC$	$\frac{CR_t}{L}$	$\frac{1}{\sqrt{LC}}\sqrt{1-\frac{R_t^2C}{L}}$	circuit in Fig. 4*a*
10	$r + sL$	R	$1/sC$	$\frac{CR_t}{L}$	$\frac{1}{\sqrt{LC}}\sqrt{1-\frac{R_t^2C}{L}}$	mirror of Fig. 4*a*

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4.2 Cross-coupled oscillators with non-balanced loads

Non-balanced load oscillators imply that $Z_2 \neq Z_3$. Two novel examples are given in Fig. 4. Consider the circuit of Fig. 4*a* where $Z_2 = R$, $Z_3 = sL + r$ and $Z_4 = 1/sC$. Using (11) we obtain the characteristic equation

$$s^2 + \left(\frac{R_t C - g_{\text{meff}} L}{LC}\right) s + \frac{1 - g_{\text{meff}} R_t}{LC} = 0 \quad (21)$$

where $R_t = R + r$. The start-up condition and oscillation frequency are thus $g_{\text{meff}} = R_t C/L$ and $\omega_0 = (1/\sqrt{LC})\sqrt{1 - (R_t^2 C/L)}$.

For the second example of Fig. 4*b*, we have $Z_2 = sL + r$, $Z_3 = R/(1 + RCs)$ and $Z_4 = 1/sC$. Using (11) then yields the third-order characteristic equation

$$s^3 + \left(\frac{1}{RC^2} + \frac{r}{LC} - \frac{g_{\text{meff}}}{C^2}\right) s^2 + \left(\frac{2 + (r/R) - g_{\text{meff}} r}{LC} - \frac{g_{\text{meff}}}{RC^2}\right) s + \frac{1 - g_{\text{meff}}(R + r)}{LRC^2} = 0 \quad (22)$$

from which the start-up condition at $r \to 0$ is $g_{\text{meff}} = RC/L$ while the oscillation frequency is $\omega_0 = 1/\sqrt{LC}$. It is clearly not necessary in any of the circuits of Fig. 4 to have two matched transistors.

The examples given in Fig. 4 are not the only possible non-balanced cross-coupled oscillators. In particular, using (11), a search for all possible Z_2, Z_3 and Z_4 which yield a positive start-up value for g_{meff} and positive ω_0 can be automated through a Matlab code. We have written this code in search for all valid cases which yield a second-order characteristic equation while using a maximum of two resistors, two capacitors or two inductors. The obtained results are given in Table 1. A similar code could be written in search for third-order cases or to impose other constraints.

4.3 Experimental and simulation results

The oscillator of Fig. 4*a* was experimentally constructed using two different Bipolar transistors of types Q2N2222 and BC109. This is to ensure that no matching or even similarity is there. The components used where $C = 50\,\text{pF}$, $L = 120\,\mu\text{H}$ (with internal resistance $r = 1.5\,\Omega$), $R = 500\,\Omega$ and $V_{CC} = 5\,\text{V}$. Oscillations started at a biasing current $I = 85\,\mu\text{A}$, which was found to be split unequally between the two transistors such that $I_a = 29\,\mu\text{A}$ and $I_b = 56\,\mu\text{A}$. Consulting (21), the ideal start-up condition would be $g_{\text{meff}} \simeq 0.2$ mA/V while oscillations actually started at $g_{\text{meff}} \simeq 0.7$ mA/V. The waveform observed at the L–C connection node is shown in Fig. 5 with a measured oscillation frequency $f_0 \simeq 1.5\,\text{MHz}$ which is close enough to the 1.9 MHz indicated by (21) given the discrete component tolerances and capacitance of the measuring probe.

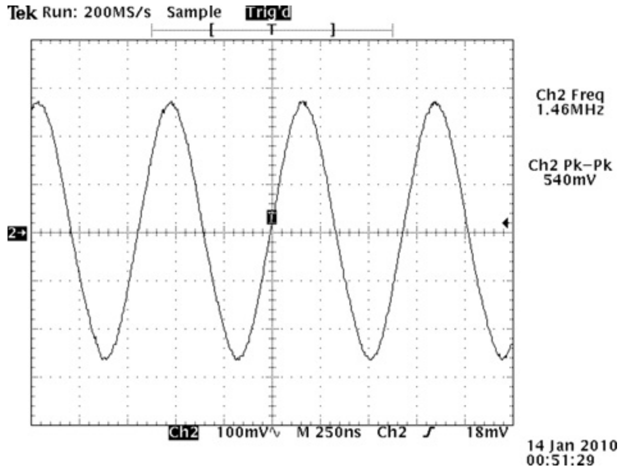

Figure 5 *Experimental waveform measured from the constructed oscillator of Fig. 4a using two non-similar Bipolar transistors*

For the third-order oscillator of Fig. 4*b*, Spice simulations using 0.25 μ CMOS process parameters were used to verify its function. We also choose to demonstrate this oscillator works with non-matched transistors by setting the aspect ratios as $(W/L)_a = (0.5\,\mu/2\,\mu)$, and $(W/L)_b = (1\,\mu/1\,\mu)$. The other components were selected to be $L = 1\,\mu\text{H}$, $r = 15\,\Omega$, $R = 1\,\text{k}\Omega$ and $C = 1$ pF. The biasing current is $I = 10\,\mu\text{A}$ and the power supply is $V_{CC} = 1.5\,V$. Fig. 6*a* shows the waveforms at the drain of M_a and also across the floating capacitor (drain voltage of M_a – drain voltage of M_b) while Fig. 6*b* shows the currents in the two transistors; which are clearly not symmetrical. The oscillation frequency is 158 MHz which is very close to the $1/\sqrt{LC}$ frequency of 159.15 MHz.

4.4 Parasitic effects

To demonstrate how parasitic effects can be incorporated into the analysis of cross-coupled oscillators using the derived two-port network equations, let us consider the general oscillator structure of Fig. 1*c*. For simplicity, we will consider the case of two matched and identically biased MOS transistors with $r_{ds} \to \infty$ and $r_s \to 0$. Then using (7) and (8) with $Z = 1/sC_{gd}$, the transmission matrix modelling the MOS transistor including the C_{gs}, C_{ds} and C_{gd} parasitics with g_m as the only design parameter becomes

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \frac{1}{1 - (g_m/sC_{gd})} \times \begin{pmatrix} 1 + \dfrac{C_{ds}}{C_{gd}} & \dfrac{1}{sC_{gd}} \\ g_m + s\left(C_{gs} + C_{ds} + \dfrac{C_{gs}C_{ds}}{C_{gd}}\right) & 1 + \dfrac{C_{ds}}{C_{gd}} \end{pmatrix} \quad (23)$$

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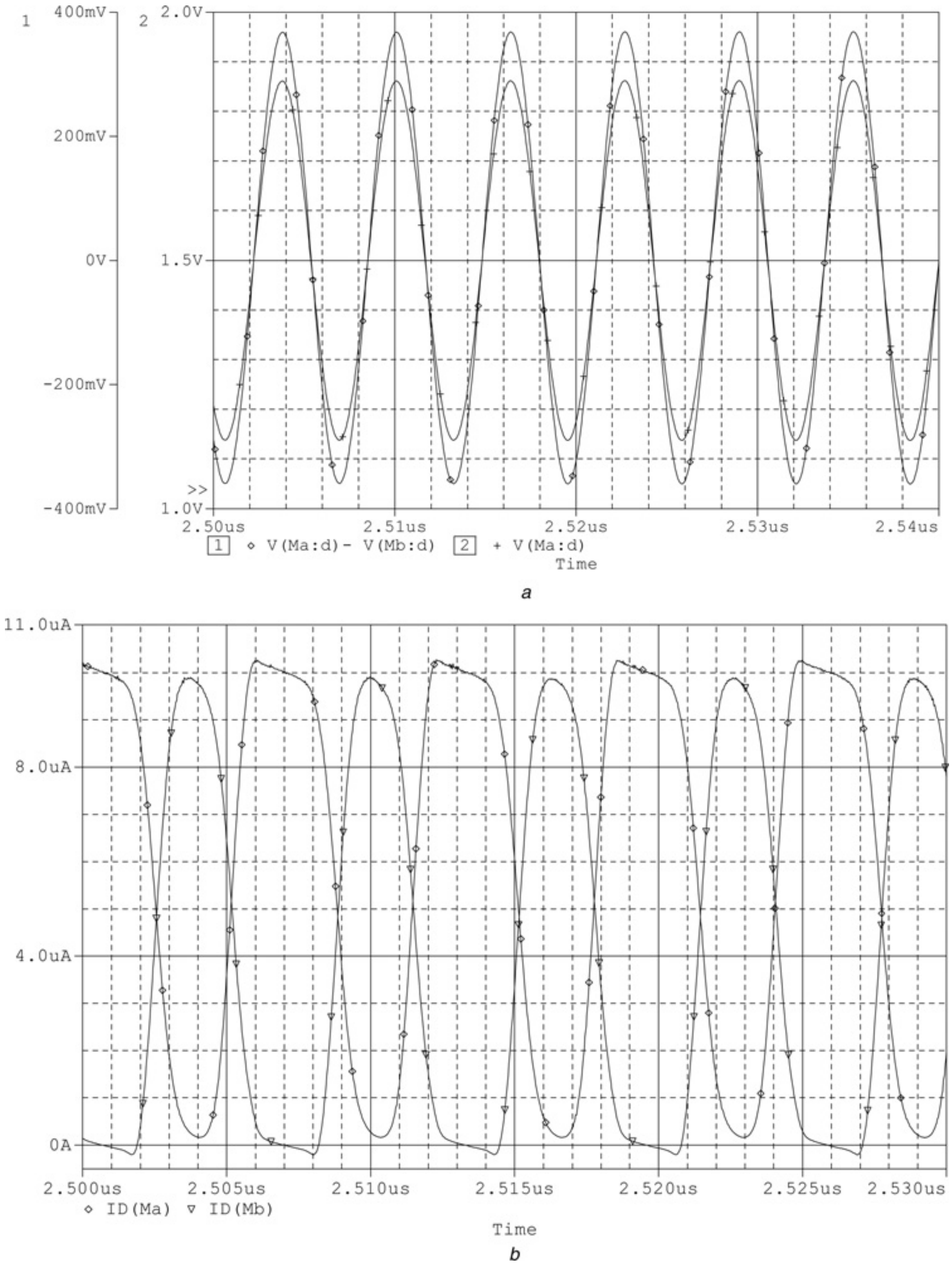

Figure 6 *Spice simulations of the non-balanced oscillator of Fig. 4b*

a Output waveform at the drain of M_a and across the floating capacitor *C*
b Drain currents in the two transistors

Substituting with the above matrix coefficients in (9) yields the characteristic equation

$$(Z_2 + Z_3)\left[\frac{2(sC_{\text{gd}} - g_m)}{Z_4} + q_2 C_{\text{gd}}^2 s^2 - q_1 g_m C_{\text{gd}} s + g_m^2\right] + 2(sC_{\text{gd}} - g_m) = 0 \qquad (24)$$

where $q_1 = 5 + (2C_{\text{ds}}/C_{\text{gd}})$ and $q_2 = 4 + 3q_1 + q_1^2 + [(q_1 - 1)C_{\text{gs}}/C_{\text{gd}}]$. Note that (24) reduces to (12) at $C_{\text{gd}} = 0$. This example emphasises the generality of the characteristic (9).

5 Conclusion

Although many researchers have targeted the analysis and design of cross-coupled oscillators, theoretical analysis using two-port network techniques was lacking. This work has addressed this issue in favour of general design equations not restricted to matched devices. It is important however to remember that two-port network analysis is a linear system analysis technique and hence the oscillation conditions derived are necessary but not sufficient conditions. In particular, some oscillators may latchup and never oscillate

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[12] and in this case one has to revert to non-linear analysis in order to explain the reason for the latchup and fix it if possible [13]. Detailed analysis of the latch-up behaviour in cross-coupled oscillators using higher-order non-linear models can be found in [14].

6 References

[1] CRANINCKX J., STEYAERT M.: 'A 1.8-GHz low-phase-noise CMOS VCO using optimized hollow spiral inductors', *IEEE J. Solid-State Circuits*, 1997, **32**, pp. 736–744

[2] JIA L., MA J., YEO K., DOI M.: '9.3-10.4 GHz band cross-coupled complementary oscillator with low phase noise performance', *IEEE Trans. Microw. Theory*, 2004, **52**, pp. 1273–1278

[3] ZHAN J., MAURICE K., DUSTER J., KORNEGAY K.: 'Analysis and design of negative impedance LC oscillators using bipolar transistors', *IEEE Trans. Circuits Syst.*, 2003, **52**, pp. 1461–1464

[4] OZOGUZ S., ELWAKIL A.S., ERGUN S.: 'Cross-coupled chaotic oscillators and application to random bit generation', *Proc. IEE Circuits Devices Syst.*, 2006, **153**, pp. 506–510

[5] YANG C.L., CHIANG Y.C.: 'Low phase-noise and low-power CMOS VCO constructed in current-reused configuration', *IEEE Microw. Wirel. Compon. Lett.*, 2008, **18**, pp. 136–138

[6] SHEKHAR S., WALLING J., ANIRUDDHAN S., ALLSTOT D.: 'CMOS VCO and LNA using tuned-input tuned-output circuits', *IEEE J. Solid-State Circuits*, 2008, **43**, pp. 1177–1186

[7] HSU M., CHIU C., CHEN S.: 'Low power design of CMOS 5-GHz voltage- controlled oscillation from narrowband to wideband switching capacitor module', *Proc. IET Circuits Devices Syst.*, 2009, **43**, pp. 99–105

[8] BUONOMO A., LO SCHIAVO A.: 'Finding the tuning curve of a CMOS-LC VCO', *IEEE Trans. Circuits Syst.*, 2008, **55**, pp. 887–891

[9] BUONOMO A.: 'Nonlinear analysis of voltage-controlled oscillators: a systematic approach', *IEEE Trans. Circuits Syst.*, 2008, **55**, pp. 1659–1670

[10] DJURHUUS T., KROZER V., VIDKJAER J., JOHANSEN T.: 'Nonlinear analysis of a cross-coupled quadrature harmonic oscillator', *IEEE Trans. Circuits Syst.*, 2005, **52**, pp. 2276–2285

[11] ELWAKIL A.S.: 'On the two-port network classification of Colpitts oscillators', *Proc. IET Circuits Devices Syst.*, 2009, **3**, (5), pp. 223–232

[12] ELWAKIL A.S., AHMED W.M.: 'On the necessary and sufficient conditions for latch-up in sinusoidal oscillators', *Int. J. Electron.*, 2002, **89**, pp. 197–206

[13] ELWAKIL A.S.: 'Explaining and eliminating latchup in a classical Wien oscillator via nonlinear design', *J. Analog Integr. Circuits Signal Process.*, 2006, **48**, pp. 239–245

[14] ELWAKIL A.S., SALAMA K.N.: 'Higher dimensional models of cross-coupled oscillators and application to design', *J. Circuits Syst. Computers*, (Special issue on 'Advances in Oscillator Analysis and Design') 2010, **19**, (4), pp. 787–799

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