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A SYSTEM AND CIRCUIT FOR GENERATING "MULTI-BUTTERFLIES"

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A system for generating a multi-butterfly chaotic attractor using the multi-level-logic pulse-excitation technique is proposed. Two-butterfly, three-butterfly and four-butterfly attractors are demonstrated. Results from an experimental setup are also shown.

Keywords: Lorenz system; butterfly chaos; chaotic oscillators.

1. Introduction

The Lorenz system which produces the well-known butterfly chaotic attractor has served as a prototype for studying chaotic behavior for a long time [Sparrow, 1982]. New systems of equations, which are not topologically equivalent to the original system but can maintain the butterfly attractor were proposed in [Chen & Ueta, 1999] and [Ueta & Chen, 2000]. The composite nature of the butterfly attractor was first explained in [Elwakil & Kennedy, 2001] and experimentally verified in [Özoguz *et al.*, 2002]. This enabled more complex butterfly architectures to be designed [Elwakil *et al.*, 2002; Elwakil *et al.*, 2003; Qi & Chen, 2006].

On the other hand, developing various techniques to generate multi-scroll chaotic attractors has received considerable interest [Lu *et al.*, 2004]. The objective is to generate more scrolls to form 1D, 2D or 3D scroll-grids, instead of the conventional double-scroll attractor, which has only two scrolls in a 1D structure. A technique was recently developed in [Elwakil & Özoguz, 2006] which is capable of generating multi-"attractors" of any type. For example, not only can a multi-"scroll" chaotic attractor be obtained, but also multi-"Rossler", multi-"Colpitts" or multi-"Duffing" chaotic attractors can be obtained where the individual "Rossler-type", "Colpitts-type" and "Duffing-type" attractors are readily famous and well-known.

The aim of this work is to show how a multi-"butterfly" attractor can be obtained by using the technique proposed in [Elwakil & Özoguz, 2006], which is basically a nonautonomous technique centered around utilizing multi-level-logic pulse exciting sources.

2. Proposed System of Equations

Consider the following novel system of differential equations

$$\dot{x} = x - y \cdot \mathrm{sgn}[f_r(t) + f_p(t) + x] \tag{1a}$$

$$\dot{y} = a \cdot \mathrm{abs}[f_p(t) + x] - by - c \tag{1b}$$

where a, b and c are constants. Two nonlinearities are involved in this system; an odd-symmetrical signum function (sgn(·)) and an even-symmetrical absolute value function (abs(·)). The system is excited via a reference periodic pulse $f_r(t)$ and a multi-level-logic periodic pulse $f_p(t)$, given respectively by

$$f_r(t) = A_r \mathrm{sgn}[\sin(\omega_r t)],$$
$$f_p(t) = \sum_{i=1}^{n} A_i \mathrm{sgn}[\sin(\omega_i t)]. \tag{2}$$

The maximum number of logic-levels of $f_p(t)$ is 2^n. In the absence of the multi-logic excitation ($f_p(t) = 0$), the above novel nonautonomous system generates a single butterfly attractor, as shown in Fig. 1(a). Since $f_r(t)$ is a reference, we fix $A_r = \omega_r = 1$. The absolute value nonlinearity is also plotted in Fig. 1(a), symmetrical with respect to $x = 0$. When $f_p(t)$ is activated, a multi-butterfly attractor with a maximum of 2^n butterflies appears. Figure 1(b) shows a two-butterfly attractor obtained when $n = 1$, $A_i = A_1 = 1/2$ and $\omega_i = \omega_1 = 1/10$. Notice that the absolute-value nonlinearity is now split into two, symmetrical with respect to $x = \pm A_1 = \pm 1/2$, as shown in Fig. 1(b). In Fig. 1(c), a three-butterfly attractor is shown. Here, $n = 2$, $\omega_1 = 1/10$, $\omega_2 = 1/5$ and $A_1 = A_2 = 1/2$. The absolute value nonlinearity is now split in three, symmetrical with respect to

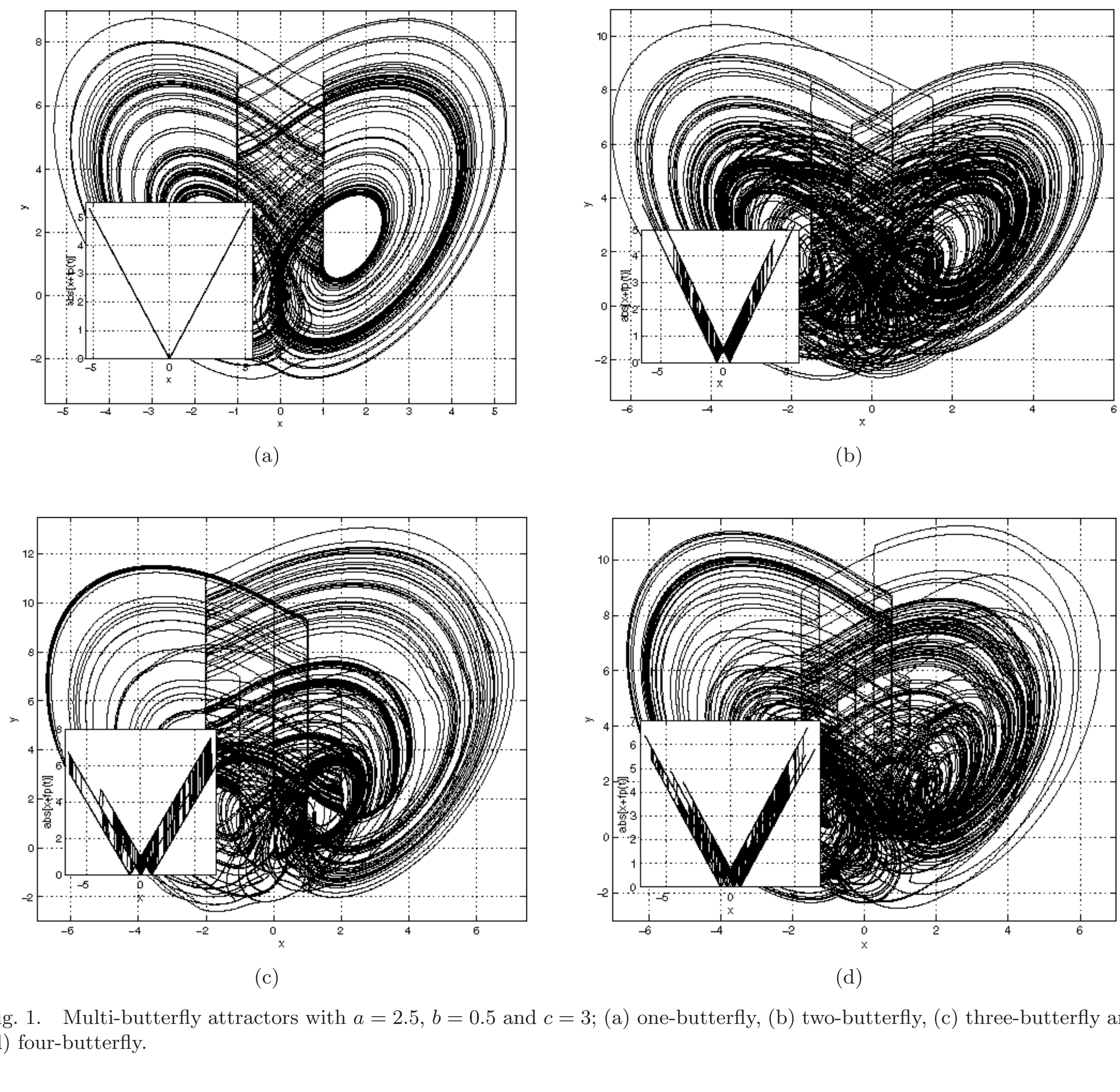

Fig. 1. Multi-butterfly attractors with $a = 2.5$, $b = 0.5$ and $c = 3$; (a) one-butterfly, (b) two-butterfly, (c) three-butterfly and (d) four-butterfly.

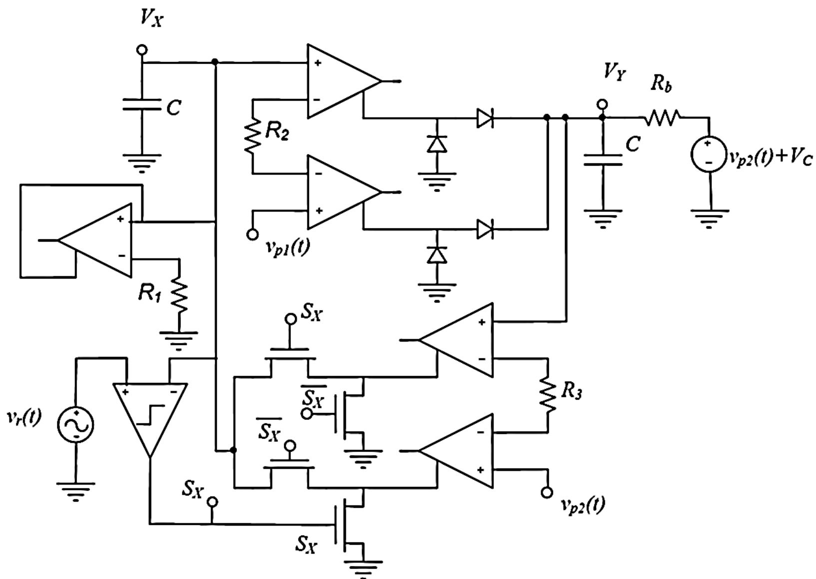

Fig. 2. Circuit for experimental verification. All op amps are AD844s. MOS transistor switches are LM4066.

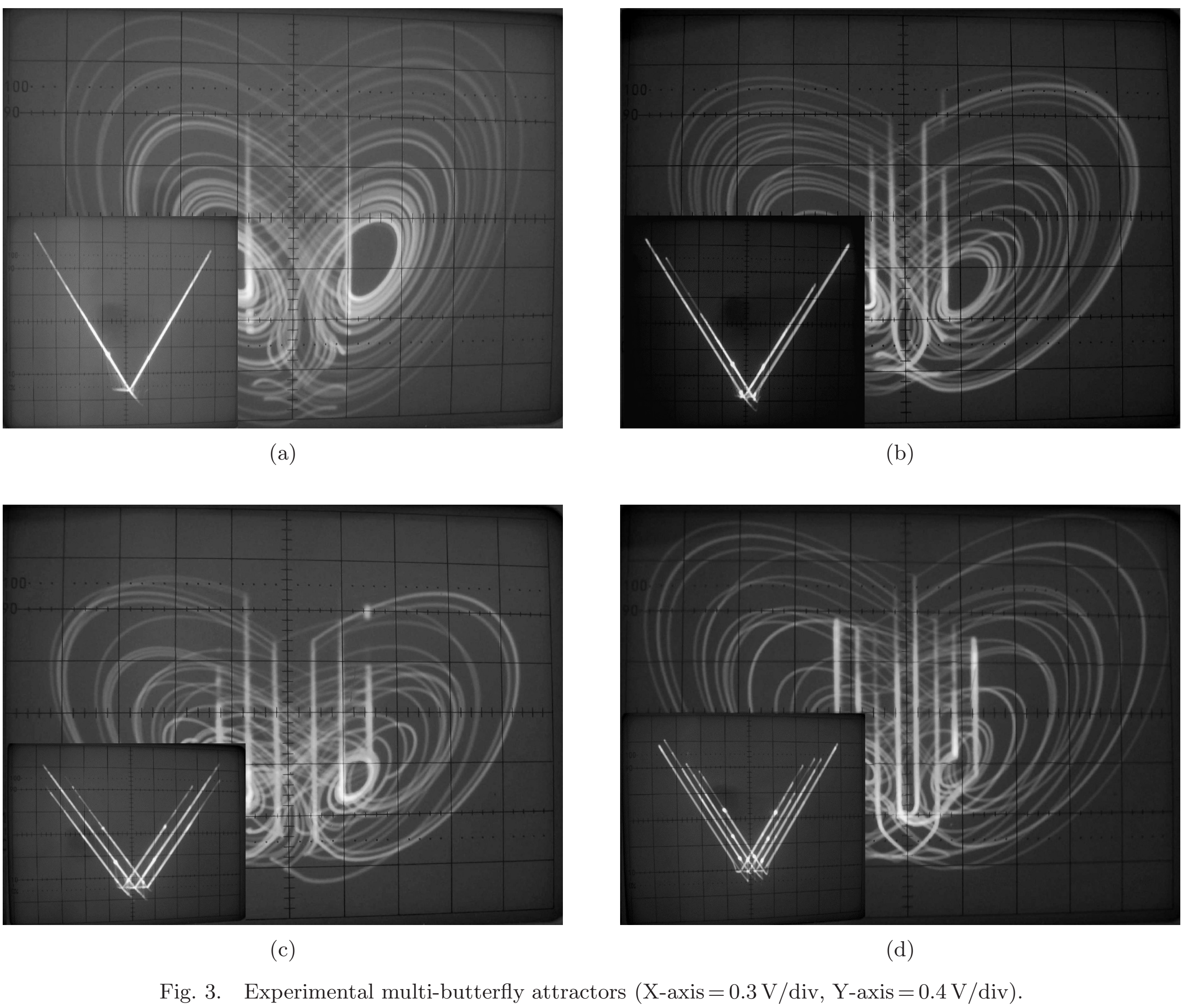

Fig. 3. Experimental multi-butterfly attractors (X-axis = 0.3 V/div, Y-axis = 0.4 V/div).

$x = \pm(A_1 \pm A_2)$; i.e. $x = 0$ and $x = \pm 1$. Finally, for $n = 2$ and choosing $A_1 \neq A_2$, a four-butterfly attractor can be obtained, as shown in Fig. 1(d) for $A_1 = 1/2$ and $A_2 = 1/4$. Evidently the even-symmetrical nonlinearity is now split in four; symmetrical with respect to $x = \pm 3/4$ and $x = \pm 1/4$.

The equilibrium points of the above system are located at $(x_0, y_0) = (\pm y_0, y_0)$ which indicates that the multi-butterfly attractor is always symmetrical around $x = 0$. It can be shown that y_0 is located at $[-c + a \cdot \sum_{i=1}^{n} \pm A_i]/(b \mp a)$ which when $n = 2$ results in eight different values if $A_1 \neq A_2$ and six different values if $A_1 = A_2$. For example, the four-butterfly in Fig. 1(d) has 16 equilibrium points (2 per wing per butterfly; 8 in each $x \lessgtr 0$ half-space) located at $y_0 = (9/16, 39/16, 19/16, 29/16)$ and $y_0 = (-3/8, -13/8, -19/24, -29/24)$. Half of these equilibrium points (i.e. eight equilibria) are virtual [Elwakil & Kennedy, 2001]. Note that it is possible to set $b = 0$ while still maintaining the multi-butterfly attractor.

It is worth noting that it is also possible to control the number of butterfly attractors by the statistical properties of the external pulse. For example, assume that $f_p(t)$ is as given by (2) for $|\mu_i(t_k)| < q$ where $\mu_i(t)$ is a real valued stationary stochastic process with zero mean, unit variance and a Gaussian probability density function ($\mu_i(t_k) = 2\pi k/\omega_i$; $k \in Z^+$). The value of the threshold q determines how often the pulses of the periodical signals are suppressed. An eight-butterfly attractor was observed with $q = 5\sigma = 5$ while a four-butterfly was obtained at $q = 0.5\sigma = 0.5$. Further reduction of q to $0.1\sigma = 0.1$ leads to a single butterfly attractor similar to that in Fig. 1(a).

3. Experimental Verification

Consider the circuit shown in Fig. 2 excited via the reference pulse voltage $v_r(t) = V_r \text{sgn}(\sin \Omega_r t)$ and the two pulse sources $v_{p_{1,2}}(t) = \sum_{i=1}^{n} V_{P_{1,2}} \times \text{sgn}(\sin \Omega_{1,2} t)$. Defining $x = V_X/V_s$, $y = V_Y/V_s$, $\tau = R_1 C$, $t_n = t/\tau$, $\omega_{1,2,r} = \tau\Omega_{1,2,r}$, $A_{1,2,r} = V_{p_{1,2,r}}/V_s$, $a = R_1/R_2$, $b = R_1/R_b$, $c = -bV_C/V_s$, where V_s is an arbitrary scaling voltage, it can be shown that this circuit realizes the set of equations:

$$\dot{x} = x - (y + f_{p_2}(t)) \cdot \text{sgn}[f_r(t) + f_{p_1}(t) + x] \quad (3a)$$

$$\dot{y} = a \cdot \text{abs}[x + f_{p_1}(t)] - b(y + f_{p_2}(t)) - c \quad (3b)$$

where $f_r(t)$ and $f_{p_{1,2}}(t)$ are as given by (2). Note that (3) reduces to (1) if $f_{p_2}(t) = 0$. However, we have chosen to show the possibility of observing a multi-butterfly using either f_{p_1} or f_{p_2}.

Fixing $C = 10\,\text{nF}$, $R_1 = 4.7\,\text{K}\Omega$, $R_2 = 2.35\,\text{K}\Omega$, $R_b = 9.8\,\text{K}\Omega$, $V_C = -0.8\,\text{V}$, $V_r = V_s = 200\,\text{mV}$, $f_r = 3400\,\text{Hz}$ and setting $v_{p_2}(t) = 0$, the multi-butterflies shown in Fig. 3 were observed. For Fig. 3(a), $v_{p_1}(t) = 0$ and only $v_r(t)$ is active. For Fig. 3(b), $v_{p_1}(t) = 50\,\text{mV} \sin 2\pi 1300t$ while for Fig. 3(c), $v_{p_1}(t) = 50\,\text{mV} (\sin 2\pi 1300t + \sin 2\pi 2000t)$. For the four-butterfly in Fig. 3(d), $v_{p_1}(t) = 50\,\text{mV} \sin 2\pi 1300t + 100\,\text{mV} \sin 2\pi 2000t$. Similar results can be obtained if $v_{p_2}(t)$ is used instead of $v_{p_1}(t)$.

4. Conclusion

A novel pulse-excited system capable of generating multi-butterfly attractors was proposed. The maximum number of butterflies increases exponentially as 2^n with the number of pulse logic levels n.

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