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Inductorless hyperchaos generator

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Abstract

A novel inductorless hyperchaos generator is proposed. The generator employs two identical linear current negative impedance converters (INICs) as the only active building blocks while nonlinearity is introduced via two generic signal diodes. The structure is truly inductorless as it is not based on active *R–C* emulation of passive inductors. A circuit implementation of the generator using current feedback op amps (CFOAs) is constructed and investigated both experimentally and using PSpice simulations. The generator is described by a fifth-order system of ordinary differential equations. Numerical simulation of the derived mathematical model is included and the positive Lyapunov exponents are estimated. © 1999 Elsevier Science Ltd. All rights reserved.

Keywords: Chaos; Hyperchaos; Current feedback op amp

1. Introduction

An increasing interest in studying the behaviour of nonlinear electronic circuits has recently developed. Of specific interest is the study of the bounded steady-state behaviour in low dimensional deterministic dynamical systems which is not an equilibrium point, not periodic and not quasi-periodic; such behaviour is termed chaos [1]. It has been shown that chaotic signals can be useful in applications such as signal encryption and communication [2,3]. Thus, many autonomous electronic circuits that generate chaos have been reported in the literature [4–10]. Most of these chaotic oscillators are modelled by a third-order system of differential equations whereas some of the recently reported oscillators are described by a fourth-order system of equations [8–10].

A chaotic signal is characterized by having a single positive Lyapunov exponent which indicates that the dynamics of the underlying chaotic attractor expand only in one direction. Whenever a chaotic attractor is characterized by more than one positive Lyapunov exponent it is termed hyperchaos. In this case, the dynamics of the chaotic attractor expand in more than one direction giving rise to a ‘‘thick’’ chaotic attractor.

In order for a circuit to exhibit hyperchaos, the minimum dimension of its state space is four. This is because one Lyapunov exponent is always zero and the sum of the exponents must be negative in order for an attractor to form [11]. Hyperchaos was first reported from computer simulations of hypothetical ordinary differential equations in [12]. The first observation of hyperchaos from a real physical system, a fourth-order electrical circuit, was later reported in [13]. Very few electronic hyperchaos generators have been reported since then [14–19]. A common feature of the circuits reported in [14–18] is the use of inductors. The only inductorless circuit proposed in [19] was based on active *R–C* emulation of the passive inductors using gyrators. Hence, the circuit is not truly inductorless.

In this work, a novel *R–C* hyperchaos generator is proposed. The generator employs two identical linear current negative impedance converters (INICs) as the active building blocks. Two general purpose diodes introduce the necessary nonlinearity required for chaos generation. A circuit realization using current feedback op amps (CFOAs) is constructed and experimentally tested. PSpice simulations of the circuit agree well with experimental observations and numerical simulations of the mathematical model describing the system. From a sampled time series of one of the state variables, two positive Lyapunov exponents are estimated.

2. The new hyperchaos generator

Fig. 1 shows the new configuration which requires four capacitors, four resistors, two diodes (which can be modelled as ideal switches) and two linear current negative impedance converters (INICs). The INIC is a two-port

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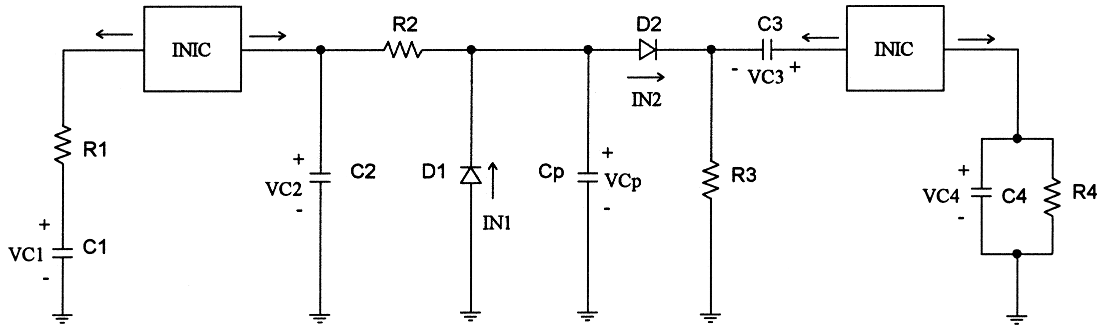

Fig. 1. The new hyperchaos generator.

network which maintains equal voltages at both input and output ports while conveying the current flowing into the input port to the output port. It is necessary to include the parasitic capacitance C_P shown in Fig. 1 in the analysis to model correctly the switching action of the diodes. This capacitor is included to model the transit capacitance associated with diode D1 (typically < 1 pF) [20]. Hence, the configuration is described by the following fifth-order set of differential equations:

$$
\begin{aligned}
C_1\dot{V}_{C1} &= \frac{V_{C2}-V_{C1}}{R_1}, \\
C_2\dot{V}_{C2} &= \frac{V_{C2}-V_{C1}}{R_1} - \frac{V_{C2}-V_{CP}}{R_2}, \\
C_3\dot{V}_{C3} &= \frac{V_{C4}-V_{C3}}{R_3} - I_{N2}, \\
C_4\dot{V}_{C4} &= \frac{V_{C4}-V_{C3}}{R_3} - I_{N2} - \frac{V_{C4}}{R_4}, \\
C_P\dot{V}_{CP} &= \frac{V_{C2}-V_{CP}}{R_2} + I_{N1} - I_{N2}
\end{aligned}
\tag{1a}
$$

where I_{N1} and I_{N2} are the nonlinear diode currents modelled respectively by:

$$
\begin{aligned}
&I_{N1} = \frac{1}{R_D}(-V_{CP}-V_\gamma),\ \text{if}\ -V_{CP} \ge V_\gamma \text{ and} \\
&I_{N1} = 0,\ \text{if}\ -V_{CP} < V_\gamma.
\end{aligned}
\tag{1b}
$$

and

$$
\begin{aligned}
&I_{N2} = \frac{1}{R_D}(V_{CP}+V_{C3}-V_{C4}-V_\gamma) \text{ if} \\
&\quad V_{CP}+V_{C3}-V_{C4} \ge V_\gamma \text{ and} \\
&I_{N2} = 0, \qquad V_{CP}+V_{C3}-V_{C4} < V_\gamma.
\end{aligned}
\tag{1c}
$$

R_D and V_γ are the diode forward conduction resistance and voltage drop respectively.

For the choice of $C_2 = C_4 = C$, $C_1 = C_3 = 2C$, $R_1 = R_2 = R_3 = R$, $R_4 = KR$ and by introducing the following dimensionless quantities:

$$\tau = \frac{t}{RC},\ X = \frac{V_{C1}}{V_\gamma},\ Y = \frac{V_{C2}}{V_\gamma},\ Z = \frac{V_{C3}}{V_\gamma},\ W = \frac{V_{C4}}{V_\gamma},\ V = \frac{V_{CP}}{V_\gamma},\ \varepsilon = \frac{C_P}{C},\ \alpha = \frac{R}{R_D},$$

equation set (1) is transformed into the following dimensionless state space representation:

$$
\begin{bmatrix} 2\dot{X} \\ \dot{Y} \\ 2\dot{Z} \\ \dot{W} \\ \varepsilon\dot{V} \end{bmatrix}
=
\begin{bmatrix}
-1 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 1 \\
0 & 0 & -(1+a_1) & 1+a_1 & -a_1 \\
0 & 0 & -(1+a_1) & 1+a_1-1/K & -a_1 \\
0 & 1 & -a_1 & a_1 & -(1+a_1+a_2)
\end{bmatrix}
\begin{bmatrix} X \\ Y \\ Z \\ W \\ V \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ a_1 \\ a_1 \\ a_1-a_2 \end{bmatrix},
\tag{2a}
$$

where,

$$
\begin{aligned}
&a_1 = \alpha, \qquad V+Z-W \ge 1 \\
&a_1 = 0, \qquad V+Z-W < 1.
\end{aligned}
$$

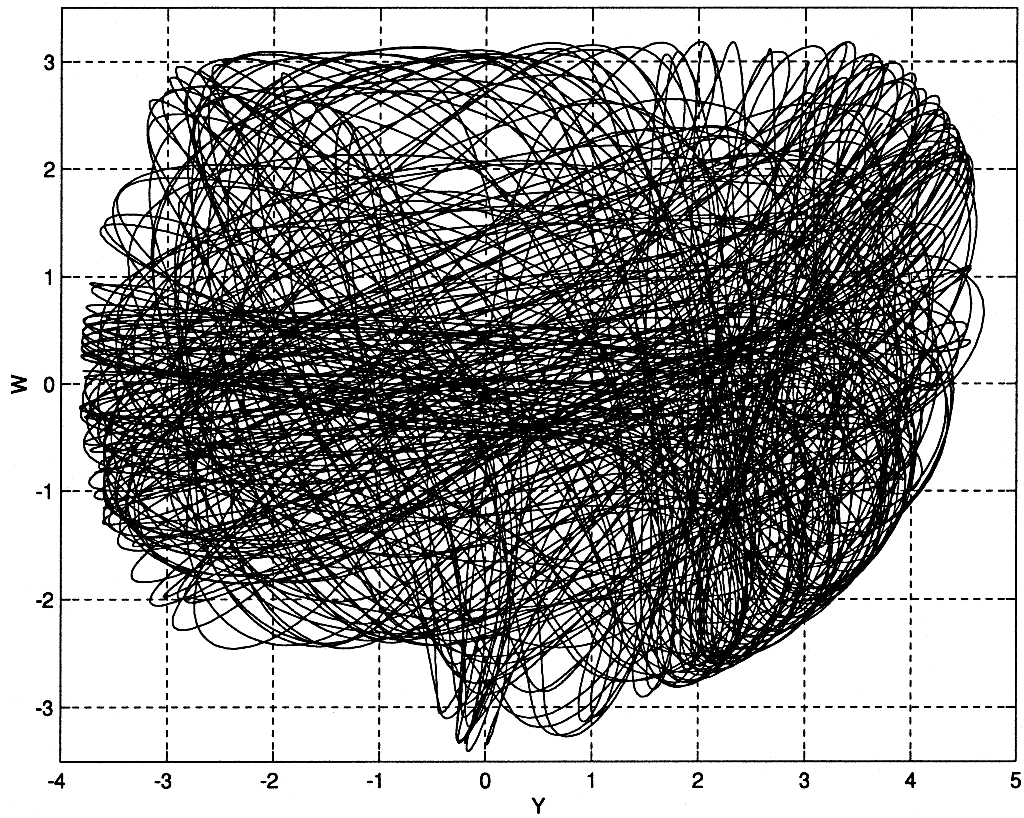

Fig. 2. Numerical simulation of the mathematical model given by Eq. (2).

and

$$a_2 = \alpha, \qquad -V \geq 1$$
$$a_2 = 0 - V < 1. \tag{2b}$$

Numerical integration of (2) using a fourth-order Runge–Kutta algorithm with a 0.0005 time step was carried out setting $\alpha = 10$, $\epsilon = 0.01$ and $K = 1.7$. The observed Y–W phase space trajectory is plotted in Fig. 2. Note that this trajectory corresponds to the V_{C2}–V_{C4} trajectory which is the best projection for observation since both capacitors are grounded and directly connected to the output ports of the INICs.

From a sampled time series of the Y state variable with 40 000 data points, the following Lyapunov exponents were estimated using the code in [21]: $+0.6706$, $+0.1103$, -0.3042, -1.188 and -4.6626. It is worth noting that the best tuning parameter for the circuit is K, which resembles a gain factor. This also enables tunability through a single grounded resistor R_4. It is also worth noting that chaotic behaviour persists for values of ϵ as large as 0.1 allowing for a physical capacitor to replace the parasitic capacitance, C_P, if desired.

3. Circuit implementation, PSpice simulations and experimental results

A circuit implementation of the configuration is shown in Fig. 3(a). The two INICs are realized using current feedback op amps (CFOAs). The CFOA is a four terminal device characterized by the following current–voltage describing matrix:

$$\begin{bmatrix} V_- \\ I_+ \\ I_C \\ V_O \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_- \\ V_+ \\ V_C \\ I_O \end{bmatrix} \tag{3}$$

indicating that the voltage applied to the noninverting input terminal (+) is transferred to the inverting input terminal (−) and the voltage at the high impedance current output terminal (C) is buffered to the voltage output terminal (O). The (+) terminal has a very high input impedance and draws negligible current while the (−) terminal has a very small input impedance. The current drawn from the (−) terminal is conveyed to the (C) terminal. In this sense, a CFOA is considered to be a cascade of a second generation current conveyor (CCII+) [22] and a buffer stage. Thus, the

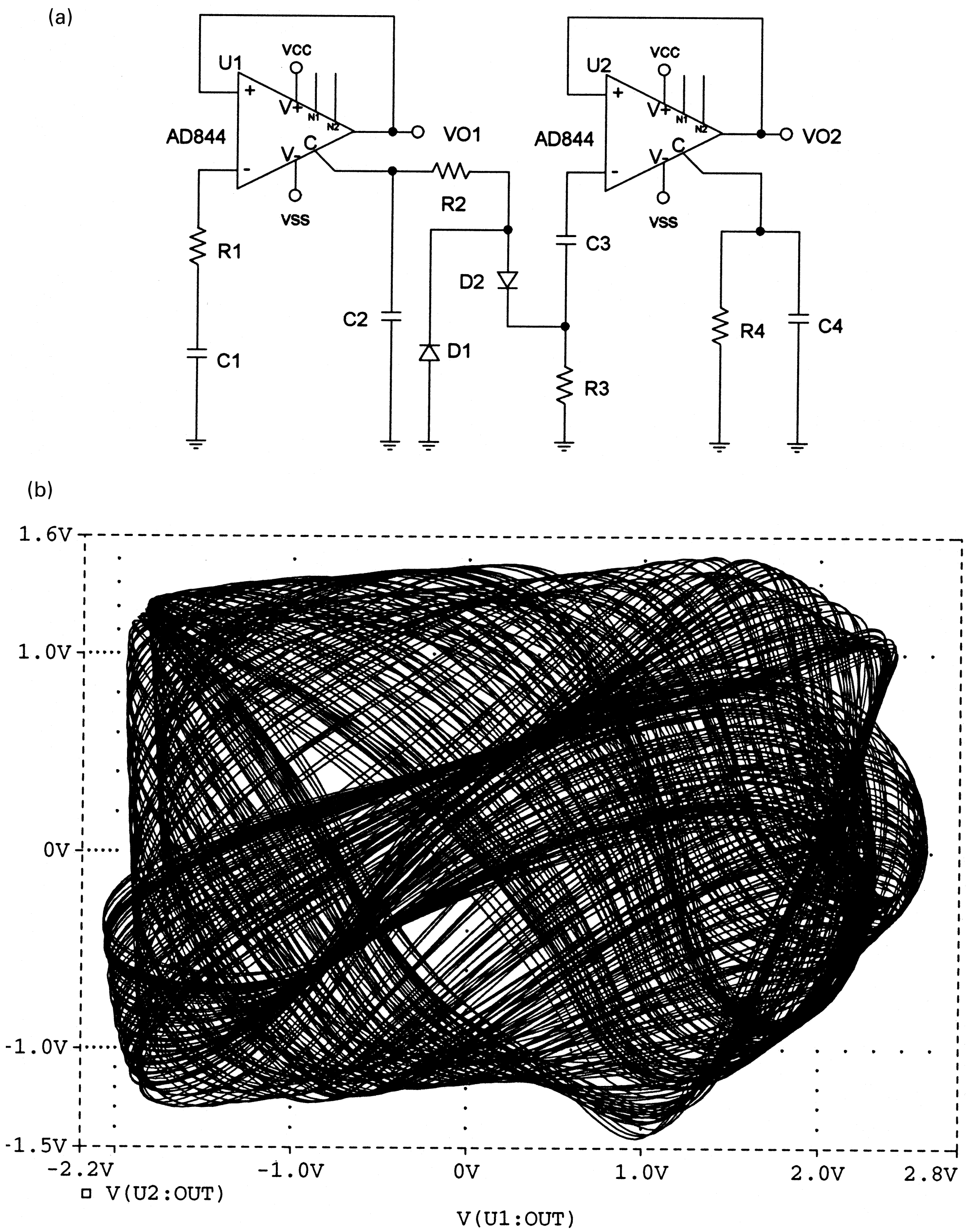

Fig. 3. (a) A circuit implementation of the hyperchaos generator using AD844 CFOAs; (b) PSpice simulation representing the V_{O1}–V_{O2} chaotic attractor.

CFOA as connected in Fig. 3(a) performs as an INIC. This realization provides two buffered signals (V_{O1} & V_{O2}) which are isolated from all circuit components; this feature facilitates measurements. Moreover, these two outputs correspond directly to the voltages across C_2 and C_4 respectively, which represent two state variables of the system.

PSpice simulations were carried out using D1N914 diodes and the following component values: $C_2 = C_4 = 50$ pF, $C_1 = C_3 = 100$ pF, $R_1 = R_2 = R_3 = 500\ \Omega$, $R_4 = 1050\ \Omega$ with ± 9 V supplies. The observed V_{O1}–V_{O2} trajectory is shown in Fig. 3(b) corresponding to the Y–W trajectory of Fig. 2.

A major advantage of this generator is its suitability for VLSI integration using any adequate implementation of an INIC. Limitations on the frequency response, power dissipation and supply voltage of the circuit will only be imposed by the designed INIC. The value of the parasitic capacitance C_P can be easily controlled in a monolithic implementation. This capacitor might also be considered as a physical capacitor with a very small value compared to the rest of the circuit capacitors.

An experimental set-up of the circuit was constructed taking: $C_2 = C_4 = 150$ pF, $C_1 = C_3 = 300$ pF, $R_1 = R_2 = R_3 = 510\ \Omega$, $R_4 = 2$ kΩ pot. (for tuning) and using D1N4148 diodes. The observed V_{O1}–V_{O2} chaotic trajectory is shown in Fig. 4.

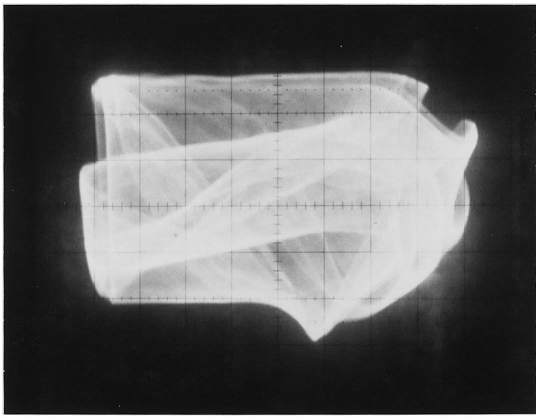

Fig. 4. Experimentally observed V_{O1}–V_{O2} trajectory. *X*-axis: 0.2 V/div; *Y*-axis: 0.2 V/div.

4. Conclusion

An inductorless hyperchaos generator suitable for VLSI integration has been designed. An implementation based on CFOAs was constructed and investigated experimentally and using PSpice. Nonlinearity in the proposed structure is introduced by two generic diodes whereas the only active elements are two identical linear INIC blocks. This decoupling of the active-linear and passive-nonlinear elements facilitates many possible implementations of the configuration depending on the circuit parameters to be optimized.

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