On the Realization of Circuit-Independent Nonautonomous Pulse-Excited Chaotic Oscillator Circuits

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***Abstract*—The aim of this paper is to present a simple circuit design method for realizing a nonautonomous chaotic oscillator given a second-order sinusoidal oscillator with two capacitors. The proposed method relies on applying a periodic pulse train, as an exciting source, and the addition of a signum-type nonlinear transconductor to the given sinusoidal oscillator. Experimental results of designed circuits are shown.**

***Index Terms*—Chaotic oscillators, nonautonomous oscillators, pulse-excited systems.**

I. INTRODUCTION

NONAUTONOMOUS chaotic oscillators are characterized by a set of nonlinear differential equations which include a time-dependent periodic driving force $f(t)$, also known as the excitation. Due to this excitation, chaos can be observed from second-order nonlinear differential equations whereas in the case of autonomous oscillators it is necessary to have a system with a minimum order of three. However, it is not really clear whether nonautonomous chaotic oscillators should actually be categorized in a separate class, at least from a practical point of view. In practice, the excitation $f(t)$ is obtained from an oscillatory circuit, which is by itself an autonomous system described at least by a second-order system of differential equations. Hence, by looking at the overall circuit implementation of a nonautonomous oscillator, including $f(t)$, one concludes that it is actually an autonomous oscillator but with a minimum order of four.

Nonautonomous chaotic oscillators introduced early in the literature [1]–[6] contained a sinusoidal-type excitation of the form $f(t) = A\sin\phi t$. It is well known that, for a linear system to sustain sinusoidal oscillations, it must be at least of second order. The simplest such system is given by

$$\ddot{Z} + \phi^2 Z = 0 \tag{1}$$

which yields $Z(t) = \sin(\phi t)$. Assuming that the nonautonomous system is originally described as $(\dot{X}, \dot{Y}) = F(X, Y, f_n(X, Y), f(t))$, where f_n is a nonlinear function, one can replace $f(t)$ with $Z(t)$ or alternatively modify the system equations to include (1) and become $(\dot{X}, \dot{Y}, \ddot{Z}) = F(X, Y, Z, f_n(X, Y))$. Such a manipulation indicates that a second-order nonautonomous system can be transformed into a fourth-order autonomous equivalent. An example of this procedure is given in Section II.

In this study, a circuit design technique, which can be used to obtain a nonautonomous chaotic oscillator from any given second-order sinusoidal oscillator with two capacitors, is presented. The technique is systematic and is based on exciting the given sinusoidal oscillator with a periodic pulse train while adding a signum-type nonlinear transconductor. These modifications result in a chaotic oscillator highly suitable for monolithic implementation. In particular, with recent trends of mixed analog/digital circuits, on the same chip, a clock signal (periodic pulse train) is always available. This signal can then be used to excite an on-chip arbitrary inductorless second-order sinusoidal oscillator with an additional binary switching-type nonlinear transconductor (effectively realized using a simple digital inverter) to result in a continuous-time chaotic oscillator synchronized with the chip clock. The chaotic output is also a pulse train. Other continuous-time chaotic oscillators that can be synchronized with a clock include switched-capacitor-based emulations, such as that in [7] and the chaotic oscillator of [8]. It is worth noting that the proposed technique has been applied to an active LC resonator circuit in [9] and that pulse-excited oscillators are receiving growing attention [10], [11].

II. PROPOSED CIRCUIT DESIGN TECHNIQUE

Consider the active linear two-port network of Fig. 1(a), characterized by being terminated at both ports with two capacitors C_1 and C_2. The short-circuit Y-parameter representation of this network is then given by

$$\begin{pmatrix} -C_1\dot{V}_{C1} \\ -C_2\dot{V}_{C2} \end{pmatrix} = \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \begin{pmatrix} V_{C1} \\ V_{C2} \end{pmatrix} \tag{2}$$

where $V_{1,2} = V_{C1,2}$, $I_{1,2} = -C_{1,2}\dot{V}_{C1,2}$, respectively, and $g_{11 \to 22}$ are constant nonzero transconductances. This network can represent an ideal sinusoidal oscillator if the condition $g_{11} = -g_{22}$ is satisfied. In addition, if the condition $g_{12}g_{21}/g_{22}^2 = -(1+n)$, where n is an arbitrary scaling factor, is satisfied, the oscillation frequency of this oscillator will be $\omega_0 = \sqrt{n}g_{22}/C$. Here, equal valued capacitors $(C_1 = C_2 = C)$ are assumed. Note that the terminal variables in the active network can always be chosen such that g_{22} and g_{21} are positive. Hence, g_{11} and g_{12} must be negative in this

Manuscript received January 3, 2004; revised March 17, 2004. This work was supported by the Turkish Academy of Sciences under the Young Scientist Award Program (İSÖ/TÜBA-GEBİP/2002-1-16) and by ITÜ Research Activity Secretariat. This paper was recommended by Associate Editor T. Saito.

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Digital Object Identifier 10.1109/TCSII.2004.836044

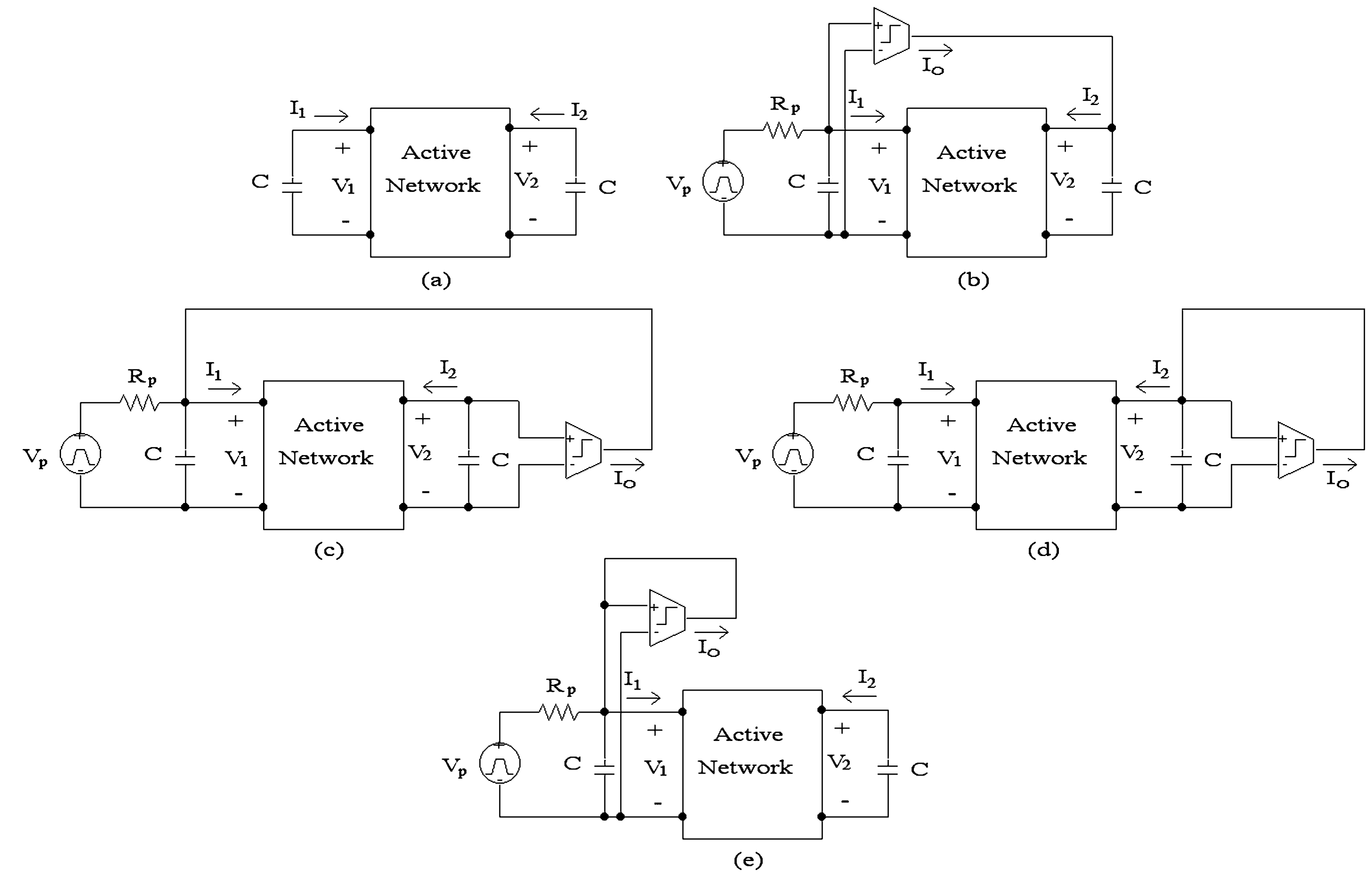


Fig. 1. Circuit-independent structures of: (a) a sinusoidal oscillator obtained by terminating an active two-port network with two equal capacitors, (b) proposed technique for realizing a pulse-excited nonautonomous chaotic oscillator by exciting the sinusoidal oscillator with a periodic pulse-train V_p and exciting the output port through a nonlinear feedforward transconductor, (c) excitation with a feedback transconductor, (d) excitation with a self feedback transconductor, and (e) excitation with a self-feedforward transconductor.

case [12]. Also note that no nonlinear amplitude stabilization mechanism of the sinusoidal oscillator is necessary to model here. A similar two-port network has also been used in [13].

Now, consider the structure shown in Fig. 1(b) where an external periodic pulse train $V_p(t)$, and a nonlinear feedforward transconductor have been added to the sinusoidal oscillator. With a peak voltage V_p and an oscillation frequency ω_p, $V_p(t)$ can be expressed as

$$V_p(t) = V_p \mathrm{sgn}(\sin \omega_p t). \tag{3}$$

A resistance R_p is used to adjust the strength of the excitation current $I_{\mathrm{ex}} = (V_p - V_1)/R_p$. The nonlinear feedforward transconductor is controlled by the port voltage V_1 and its output current I_o is given by

$$I_o = I_{\mathrm{sat}} \mathrm{sgn}(V_1) = \begin{cases} I_{\mathrm{sat}}, & V_1 \geq 0 \\ -I_{\mathrm{sat}}, & V_1 < 0. \end{cases} \tag{4}$$

I_{sat} is the saturation current of the transconductor. Using (2)–(4), one can then write the state-space equations describing Fig. 1(b) as

$$C\dot{V}_{C1} = -(g_{11} + g_p)V_{C1} - g_{12}V_{C2} + g_p V_p(t) \tag{5a}$$
$$C\dot{V}_{C2} = -g_{21}V_{C1} - g_{22}V_{C2} + I_o \tag{5b}$$

where $g_p = 1/R_p$.

By introducing the dimensionless variables $X = -(g_{22}^2/g_{12})(V_{C1}/I_{\mathrm{sat}})$, $Y = g_{22}(V_{C2}/I_{\mathrm{sat}})$, $\varepsilon = g_p/g_{22}$, $\beta = -(g_p g_{22}/g_{12})(V_p/I_{\mathrm{sat}})$, $\phi = \omega_p(C/g_{22})$, and $\tau = t(g_{22}/C)$, the above system of equations transforms into

$$\dot{X} = (1 - \varepsilon)X + Y + \beta p(\tau) \tag{6a}$$
$$\dot{Y} = -(1 + n)X - Y + \mathrm{sgn}(X) \tag{6b}$$
$$p(\tau) = \mathrm{sgn}(\sin \phi\tau) = \begin{cases} 1, & \sin\phi\tau \geq 0 \\ -1, & \sin\phi\tau < 0 \end{cases} \text{ and } \mathrm{sgn}(X) = \begin{cases} 1, & X \geq 0 \\ -1, & X < 0. \end{cases} \tag{6c}$$

The numerical simulation result of this model using a fourth-order Runge–Kutta algorithm with adaptive step size is shown in Fig. 2(a) for the parameter set $(\varepsilon, n, \beta, \phi) = (0.05, 1, 0.5, 0.2)$. The observed attractor is similar to a four-scroll attractor with the equilibrium points $(x_0, y_0) = (1/n)[(\pm\beta\pm1), \mp(1+n)\beta\mp 1]$ for $\varepsilon \to 0$. Corresponding to Fig. 2(a), these equilibria are located at $(1.5, -2)$, $(-1.5, 2)$, and $(\pm 0.5, 0)$ and can be identified in the figure.

Now recalling (1), it is possible to transform this second-order nonautonomous system into a fourth-order autonomous system by replacing $p(\tau)$ with $\mathrm{sgn}(Z)$. The same chaotic attractor is observed in this case and it becomes possible to construct the

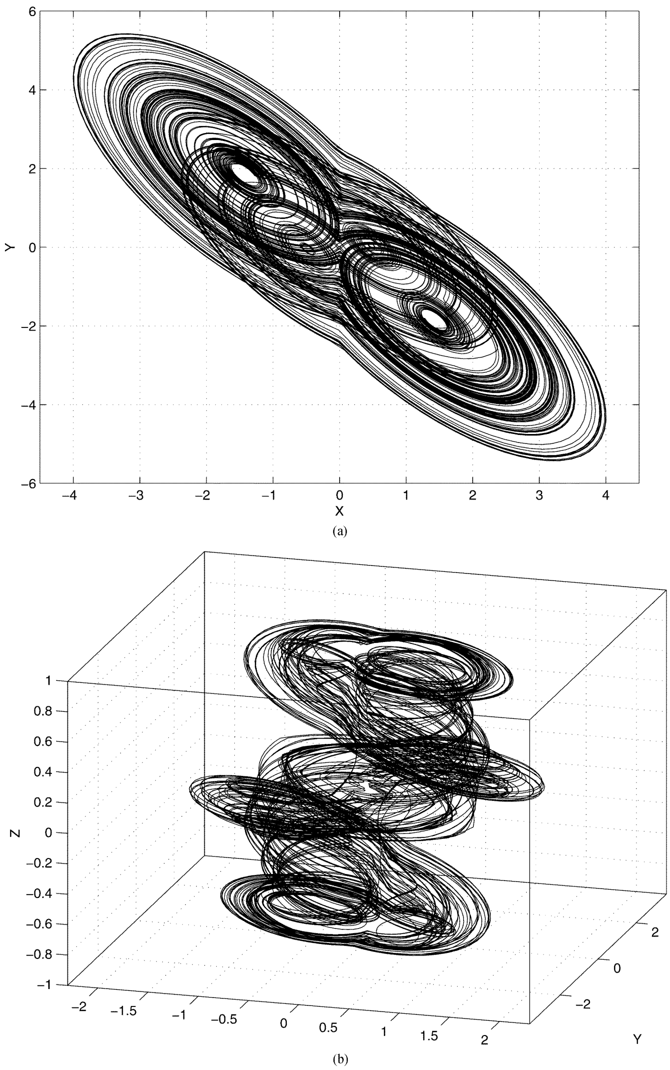


Fig. 2. Chaotic attractor obtained from (6) with $(\varepsilon, n, \beta, \phi) = (0.05, 1, 0.5, 0.2)$. (a) Projection in the X-Y plane and (b) constructed 3-D view by using (1) in conjunction with (6).

three-dimensional (3-D) X-Y-Z view of the attractor, shown in Fig. 2(b).

III. On the Mechanism of Chaos Generation

It is well known that electronic chaotic oscillators are dissipative systems with strange attractors as steady-state solutions. Melnikov's conditions can be used to show the existence of horseshoes in nearly Hamiltonian forced planar systems. In particular, and given a planar perturbed nonlinear system of the form $\dot{X} = f(X) + \varepsilon g(X, \tau)$, where f and g are smooth functions and g is also periodic in time with period T, it can be shown that chaotic motions and horseshoes, as implied by the Smale–Birkhoff Homoclinic Theorem, require that [14], [15]:

1) for $\varepsilon = 0$, the system is Hamiltonian and has a homoclinic orbit passing through a saddle-type critical point;
2) for $\varepsilon = 0$, the system has one parameter family of periodic orbits $\theta_\gamma(t)$ of period T_γ on the interior of the homoclinic orbit with $(\partial\theta_\gamma(0)/\partial\gamma) \neq 0$;

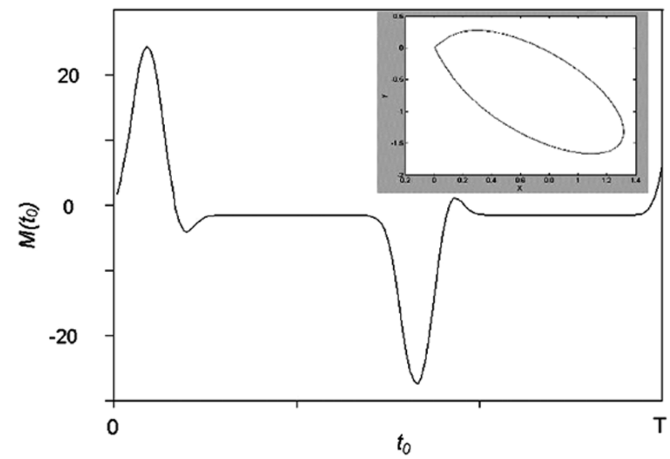


Fig. 3. Zeros of the Melnikov function calculated on the homoclinic orbit shown in the upper left corner for $T = 10\pi$.

3) the Melnikov function $M(\tau_0)$ defined as $M(\tau_0) = \int_{-\infty}^{\infty} f^0(\tau) \wedge g^0(\tau + \tau_0) d\tau$ has simple zeroes for $\tau_0 \in [0, T]$. The wedge product is evaluated on the homoclinic loop of the system which indicates transversal interconnections of the stable and unstable manifolds of the saddle-type fixed point of the Poincare map. Details of this chaos generating mechanism can be found in [15].

The system (6) can be rewritten as

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -(1+n) & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} \varepsilon g_1(X, Y, \tau) \\ g_2(X, Y) \end{pmatrix} \quad (7)$$

where $g_1(X, Y, \tau) = g_1(X, \tau) = -X + (\beta/\varepsilon)p(\tau)$ and $g_2(X, Y) = g_2(X) = \mathrm{sgn}(X)$.

It can be shown that the unperturbed ($\varepsilon \to 0$) part of the system has a homoclinic orbit and it corresponds to a Hamiltonian center system. Replacing the nonsmooth $g_2(X) = \mathrm{sgn}(X)$ nonlinearity with a smooth approximation of the form $g_2(X) = \tanh(kX)$ with a large value for k (e.g., $k = 10$) and similarly replacing the nonsmooth $p(\tau) = \mathrm{sgn}(\sin\phi\tau)$ in $g_1(X, \tau)$ with the approximation $p(\tau) = \tanh(k \sin\phi\tau)$, it was numerically verified that the Melnikov function given below, calculated on the homoclinic orbit of (7), has simple zeros for $\tau_0 \in [0, T]$, as shown in Fig. 3 for $T = 10\pi$ as follows:

$$M(\tau_0) = \int_{-\infty}^{\infty} [(1+n)X + Y - \tanh(10X)] \cdot \left[-X + \frac{\beta}{\varepsilon}\tanh(10\sin\phi(\tau + \tau_0))\right] d\tau. \quad (8)$$

Therefore, it can be deduced that the system in (7) satisfies the conditions above for chaos generation.

IV. Alternative Configurations

There are three straightforward alternative configurations which can be deduced form that of Fig. 1(b) by changing the location of the nonlinear transconductor. These configurations are shown in Fig. 1(c)–(e), respectively. All configurations can be modeled by (7) with different values for $g_1(X, Y, \tau)$ and $g_2(X, Y)$, as summarized in Table I. A chaotic attractor

TABLE I
DIFFERENT VALUES OF g_1 AND g_2 IN (7) CORRESPONDING TO FIG. 1(C)–(E) ($\alpha = g_{11}/g_{12}$)

Figure	$g_1(X,Y,\tau)$	$g_2(X,Y)$
1(c)	$-X + \frac{\beta}{\varepsilon}p(\tau) + \frac{\alpha}{\varepsilon}\text{sgn}(Y)$	0
1(d)	$-X + \frac{\beta}{\varepsilon}p(\tau)$	sgn(Y)
1(e)	$-X + \frac{\beta}{\varepsilon}p(\tau) + \frac{\alpha}{\varepsilon}\text{sgn}(X)$	0

similar to that of Fig. 2 can be observed from the model corresponding to Fig. 1(d), for example, using the parameter set $(\varepsilon, n, \beta, \phi) = (0.05, 1, 3, 0.2)$.

It is interesting to note that (7), when corresponding to the structure of Fig. 1(c) (see Table I), can be rewritten in the form

$$\ddot{Y} + \varepsilon\dot{Y} + (\varepsilon + n)Y + \alpha(1+n)f(Y) = -\beta(1+n)p(\tau) \quad (9)$$

where $f(Y) = \text{sgn}(Y)$ and $\alpha = g_{22}/g_{12}$. Now if $f(Y)$ is a cubic nonlinearity of the form $f(Y) = Y^3 - Y$ and a sinusoidal excitation is used instead of the pulse excitation, i.e., $p(\tau) = A\sin\phi\tau$, the well-known Duffing oscillator is obtained [16]. Choosing $n = 1$ and $\alpha = \beta = 1/2$ in this case, the above equation simplifies to

$$\ddot{Y} + \varepsilon\dot{Y} + \varepsilon Y + Y^3 = -A\sin\phi\tau. \quad (10)$$

A typical Duffing attractor in the $Y - \dot{Y}$ phase space can be observed with $\varepsilon = 0.01$ and $\phi = 0.2$.

It is also interesting to note that (7), when corresponding to the structure of Fig. 1(d) (see Table I), can be rewritten in the form

$$\ddot{Y} + (\varepsilon - f^{\backslash}(Y))\dot{Y} + (\varepsilon + n)Y + (1-\varepsilon)f(Y) = -\beta(1+n)p(\tau))$$

where $f^{\backslash}(Y) = df(Y)/dY$. Now if $f(Y)$ is chosen to be $f(Y) = \varepsilon Y^3/3$ and $p(\tau) = A\sin\phi\tau$ with $n = 1 - \varepsilon$ and $\beta = 1/(2-\varepsilon)$, the above equation will reduce to a form close to the classical forced van der Pol oscillator [16] and will exhibit similar chaotic behavior. The same also applies to the structure in Fig. 1(e). It is thus seen that, by changing the type and position of the excitation and/or nonlinear transconductor, the structures of Fig. 1 can be flexibly used to realize a variety of nonautonomous dynamical systems.

V. CIRCUIT DESIGN EXAMPLES

Two chaotic oscillator circuits are given here as examples of the proposed technique. A voltage comparator and a current feedback op amp (CFOA) are used to realize the nonlinear transconductor element, as shown in Fig. 4(a). This realization offers both voltage and current outputs. In this case, the saturation current I_{sat} equals V_{cc}/R_f, where V_{cc} is the dc power supply of the comparator. In the following, all active devices are supplied with $V_{cc} = 5$ V.

The first design example is shown in Fig. 4(b) and is based on the single-resistance-controlled oscillator proposed in [17] and marked within the dashed box. By defining $X = V_1/((R_3/R_2) + 2(R_3/R_4))R_3I_{\text{sat}}$, $Y = V_2/R_3I_{\text{sat}}$,

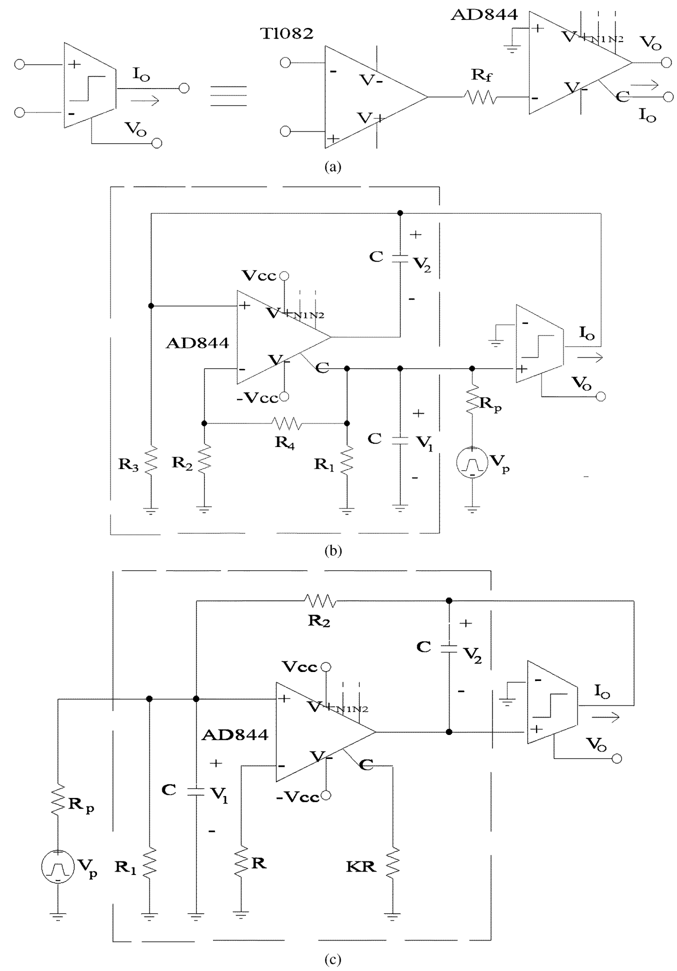


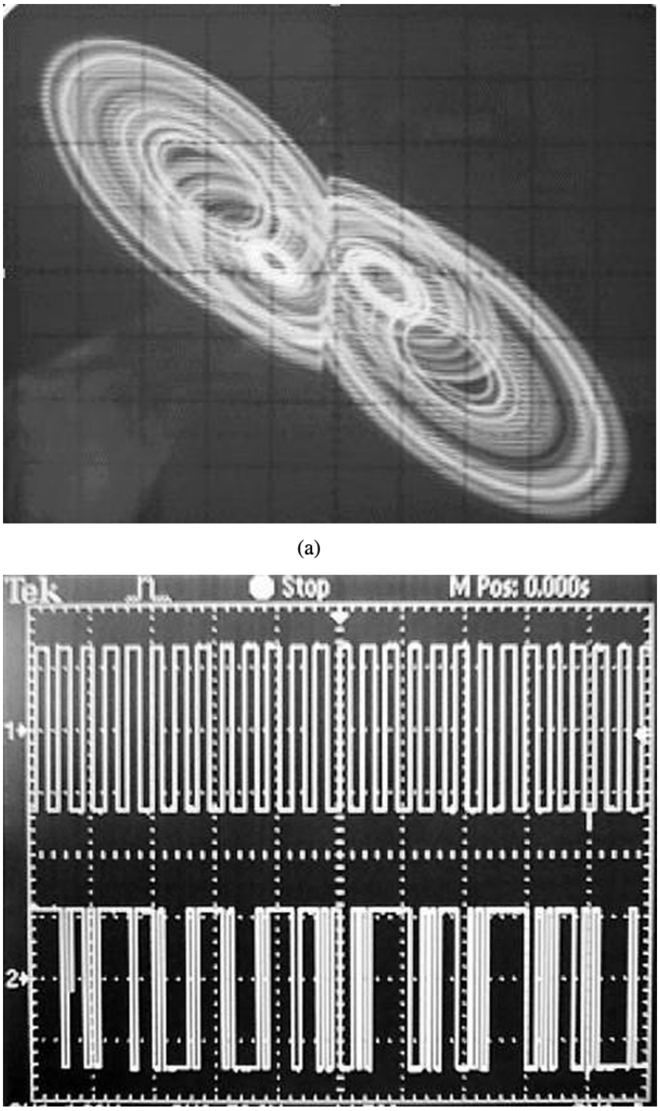
Fig. 4. Circuit realizations of (a) the nonlinear transconductor, (b) a chaotic oscillator based on the single resistance controlled oscillator of [17], and (c) chaotic oscillator based on the Wien bridge oscillator.

and $\tau = t/R_3C$, the general state equations (6) can describe this chaotic oscillator with $\varepsilon = 1 + (R_3/R_1) + (R_3/R_p) - (R_3/R_2)$, $\beta = (V_p/I_{\text{sat}}R_p(R_3/R_2 + 2R_3/R_4))$, $n = (R_3/R_2) + 2(R_3/R_4) - 1$, and $\phi = \omega_pR_3C$.

The circuit was tested taking $R_1 = R_3 = 10$ kΩ, $R_2 = 4020$ Ω, $R_4 = 100$ kΩ, $R_p = 20$ kΩ, $R_f = 125$ kΩ and $C = 1$ nF. The periodic pulse excitation frequency $f_p = \omega_p/2\pi = 2.8$ kHz and its amplitude $V_p = 1$ V. These component values correspond to $\varepsilon = 0.012$, $\beta = 0.465$, $n = 1.68$, and $\phi = 0.2$. The observed $V_1 - V_2$ phase-space trajectory is shown in Fig. 5(a).

In Fig. 4(c), a second design example based on the classical Wien bridge oscillator, which employs a noninverting amplifier with gain K, is shown. By defining $X = V_1/R_2I_{\text{sat}}$, $Y = V_2/R_2I_{\text{sat}}$ and $\tau = t/R_2C$, this chaotic oscillator can also be described using the general form of (6) with $\varepsilon = 2 + (R_2/R_1) + (R_2/R_p) - K$, $\beta = (V_p/I_{\text{sat}}R_p)$, $n = K - 2$ and $\phi = \omega_pR_2C$.

The circuit was constructed with $R_1 = R_2 = 10$ kΩ, $R_f = 130$ kΩ, $R_p = 50$ kΩ, $K = 3$, $f_p = 3050$ Hz and $V_p = 1$ V. These values correspond to $\varepsilon = 0.2$, $\beta = 0.5$, $n = 1$, and $\phi = 0.2$, respectively. The observed $V_1 - V_2$ phase projection is similar to that in Fig. 5(a) and a sample of the periodic pulse-input and chaotic pulse-output waveforms is shown in Fig. 5(b).



(a)

(b)

Fig. 5. Experimental observations representing (a) the $V_{C1} - V_{C2}$ phase-space projection for the circuit in Fig. 4(b) (X axis:0.5 V/div, Y axis:0.5 V/div) and (b) sample of the periodic input and chaotic output pulse time waveforms for the circuit in Fig. 4(c).

VI. Conclusion

In this paper, a simple circuit design method, which allows the systematic derivation of nonautonomous chaotic oscillators from any given sinusoidal oscillator with two capacitors, was presented. The resulting oscillators are directly compatible with digital interfaces and can be considered as periodic-to-chaotic clock converters.

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