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On the necessary and sufficient conditions for latch-up in sinusoidal oscillators

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Two necessary and sufficient conditions for the occurrence of dynamical latch-up in sinusoidal oscillators are proposed. These conditions result from studying the stability of the equilibrium points associated with each region of operation of the fundamentally nonlinear driving-point characteristics that are necessary to model accurately the behaviour of sinusoidal oscillators. The fact that an equilibrium point can be either virtual or real is the key to predicting latch-up. We numerically validate our proposal on the derived models of some well-known oscillators by predicting and confirming either the possibility or the impossibility of latch-up.

1. Introduction

Sinusoidal oscillators are important circuit elements for many systems. Characterizing their performance is therefore crucial. Despite the many contributions of various researchers, based on a linear circuit theory perspective (Wojtyna and Borys 1986, Borys 1987, Heurtas *et al*. 1990, Hajimiri and Lee 1998), some of the most important characteristics of oscillators, such as the exact amplitude and frequency of oscillation and their stability, cannot be accurately predicted by using linear analysis techniques. Furthermore, using linear design techniques, some oscillators never oscillate as intended. Instead, they either exhibit latch-up, whereby their output remains indefinitely at a fixed operating point or exhibit hysteresis behaviour. In order to predict such dynamical behaviours, an accurate nonlinear model of the oscillator must be derived (Guckenheimer and Holmes 1983, Bonatti *et al*. 1999). In particular, it is well known that a stable limit cycle cannot be sustained in a linear system since perturbations which might cause one or more of its eigenvalues to move into the right-half plane will automatically cause trajectories to diverge. The existence of a nonlinear driving point characteristic, which can guarantee that after a set of break points a stable equilibrium point with all its eigenvalues in the left-half plane is reached, is a necessary condition to sustain the limit cycle. However, this condition is not sufficient, as will be discussed in the following.

In this work, we propose two necessary and sufficient conditions for latch-up to occur in sinusoidal oscillators. We apply these conditions to the classical second-order voltage-controlled negative resistance oscillator and the third-order less-than-unity gain oscillator, and successfully detect occurrence of latch-up. Furthermore, we show that the Wien-bridge and the phase-shift oscillators cannot by nature exhibit latch-up. The latch-up behaviour was studied by Martinez *et al*. (1989, 1990, 1991)

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from a linear system perspective following Loeb's criterion for stability of the limit cycle. However, the conditions proposed by Martinez *et al.* are necessary but not sufficient and apply mainly for op amp-based oscillators. In this work, we approach the problem from a nonlinear dynamics perspective, treating oscillators as strong nonlinear systems. We show that it is important to consider the behaviour of the system at all equilibrium points, not only the origin.

2. Proposed conditions

An autonomous continuous-time dynamical system is defined by a system of ODEs (state equations) of the form $\dot{x} = f(x) + b$, where the vector field f is independent of time. An equilibrium point is a state x_e at which $\dot{x} = 0$. The eigenvalues of $f(x)|_{x \to x_e}$ define whether x_e is stable or unstable. If the real parts of all eigenvalues are negative, x_e is *stable* and is called a sink (Chua *et al.* 1987). If any eigenvalue has a positive real part, x_e is *unstable*. If all eigenvalues have positive real parts, x_e is called a source.

An electronic circuit can have different regions of operation each of which has its own equilibrium point. If an equilibrium point physically lies inside the corresponding region of operation, it is said to be *real*; if it lies outside this region it is said to be *virtual* (Chua *et al.* 1987).

Most sinusoidal oscillators employ active devices (e.g. op amps) with saturation-type odd-symmetrical voltage transfer characteristics. Such characteristics can be modelled by a three-segment piecewise-linear approximation; an inner segment and two outer segments. Hence, in general, three equilibrium points exist. The origin $x_e = 0$ is always a *real* equilibrium point in the inner segment. Based on the Barkhausen criterion, the condition for oscillation, derived from linear analysis, guarantees that this equilibrium point becomes *unstable* (critically stable) with a pair of pure imaginary eigenvalues. This condition is necessary but not sufficient to admit sinusoidal oscillations (or near sinusoidal oscillations when the complex pair is located in the right-half plane†). To obtain a set of necessary and sufficient conditions, the following must also be satisfied:

(1) The equilibrium points in the outer segments of the nonlinearity must not be both *real* and *stable*.

(2) If the oscillator is described by a third-order system of ODEs, then the real eigenvalue associated with the equilibrium point at the origin must remain negative.

(3) The proper state variable(s) are chosen as the independent control variable(s) for the nonlinearity such that hysteresis is not stimulated (Kennedy and Chua 1991, Elwakil 2000, Elwakil and Kennedy 2000).

If condition (3) is not carefully considered, a relaxation, rather than sinusoidal, oscillator will result. Note that a system will settle indefinitely at an equilibrium point if and only if it is both *real* and *stable*. Therefore latch-up will occur when:

(1) any of the equilibrium points in the outer segments of the nonlinearity becomes both *real* and *stable,* and

† The amount of distortion resulting from shifting the complex conjugate eigenpair into the right-half plane is quantified via the total harmonic distortion (THD).

(2) the equilibrium point at the origin acquires a *positive real* eigenvalue.

In the following sections, we study the nonlinear models of a collection of sinusoidal oscillators and investigate the possibility of latch-up.

3. The active tank resonator

Consider the passive bandpass filter shown in figure 1 (*a*). The Barkhausen condition for oscillation implies that $R \rightleftarrows \infty$. The result is an ideal tank circuit that maintains a sinusoidal oscillation with an arbitrary amplitude defined by the initial conditions. In practice, there exists a small resistance r_L in series with the inductor and therefore any oscillation excited will eventually die.

In order to sustain an oscillation, the *passive* tank must be transformed into an *active* tank. One method for so doing is shown in figure 1 (*b*). It is straightforward to show that the condition for oscillation in this network is given by $K = 1 + r_L RC/L$ and the frequency of oscillation is $\omega_0 = (1/\sqrt{LC})\sqrt{1-(K-1)r_L/R}$.

A widely used alternative to activate the tank is shown in figure 1 (*c*). Here, a negative resistor $-r$ is placed in parallel with the passive tank. The condition for oscillation is found to be: $r = L/Cr_L$ and the frequency of oscillation is $\omega_0 = (1/\sqrt{LC})\sqrt{1 - r_L/r}$. Note that the activation method of figure 1 (*b*) is actually equivalent to placing a negative resistor $-r = -R/(K-1)$ across the passive tank. Hence, both methods are essentially the same. The ideal lossless tank where $r_L = 0$ corresponds to $K = 1$ and $r = \infty$.

Circuit realizations of negative resistors are either current- or voltage-controlled. Of course, in figure 1 (*c*), the negative resistor as it is placed must be voltage-controlled, otherwise hysteresis will be stimulated (Kennedy and Chua 1991). A classical voltage-controlled negative resistor circuit is shown in figure 2 along with its equivalent amplifier-based model. This circuit realizes a negative resistance $-r = -R_h R_1/R_2$ (recall that $K = 1 + R_2/R_1$) so long as $|V_i| < V_{sat}/K$. When V_i exceeds $\pm V_{sat}/K$, the output voltage of the amplifier saturates, remaining at $\pm V_{sat}$. The input resistance to the network in this case is R_h.

A PSpice simulation of these negative resistor nonlinear characteristics is shown in figure 3 for three different values of R_1. The range over which the circuit behaves

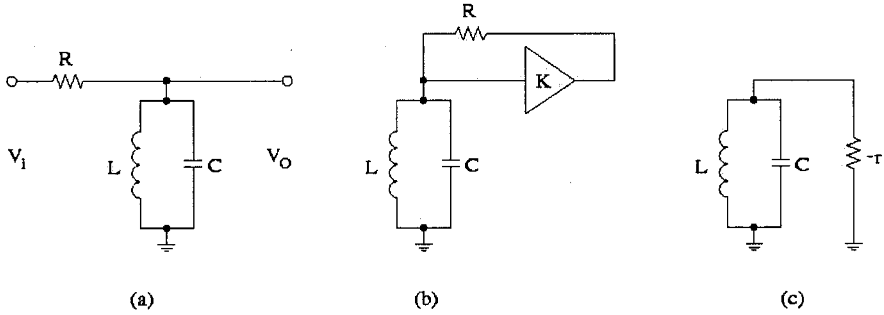

Figure 1. Passive and active tank resonators: (*a*) a band-pass filter transforms into a passive tank circuit when $R \rightleftarrows \infty$; (*b*) tank circuit activated by an amplifier; (*c*) tank circuit activated by a negative resistor.

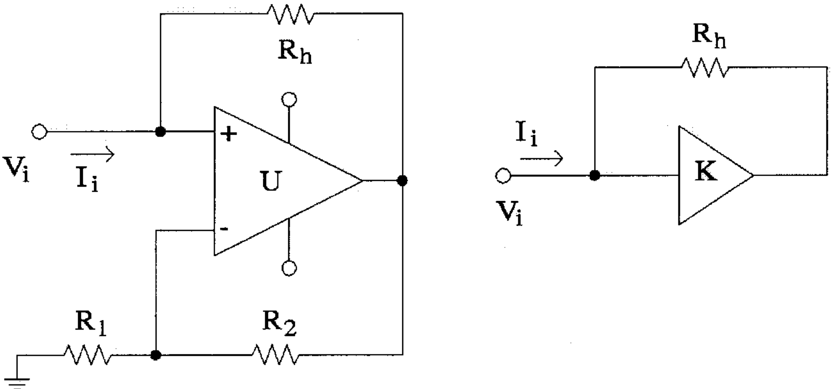

Figure 2. Voltage-controlled negative resistor and its equivalent amplifier-based model.

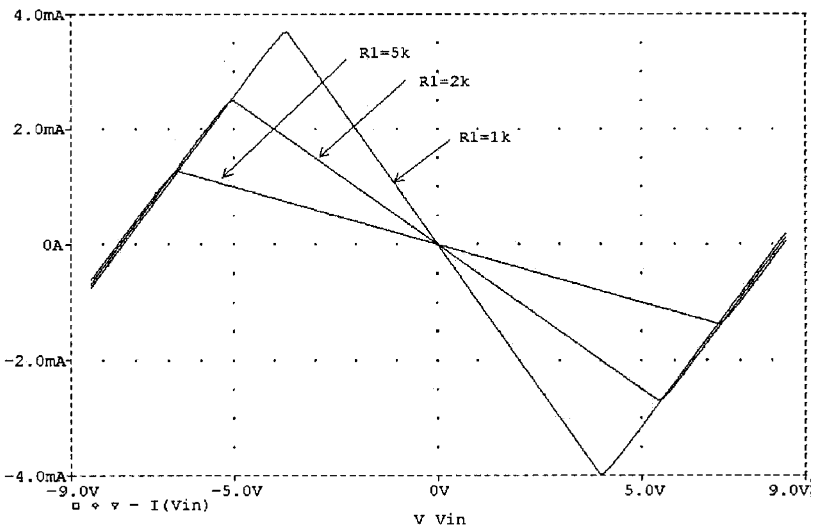

Figure 3. V–I characteristic of the voltage-controlled negative resistor ($R_h = R_2 = 1\,k\Omega$, the op amp is AD712 biased with ± 9 V supplies).

as a linear negative resistor can be extended by bringing the amplifier gain K as close as possible to unity.

The activated tank circuit configuration of figure 1 (c) is modelled by

$$C\dot{V}_C = -I_L - I(V_C) \tag{1a}$$

$$L\dot{I}_L = V_C - r_L I_L \tag{1b}$$

and $I(V_C)$ is the current in the voltage-controlled negative resistor, modelled as (see figure 3)

$$I(V_C) = I_{\mathrm{BP}} \left\{ \begin{array}{cc} \dfrac{V_0 + V_C}{V_0 - V_{\mathrm{BP}}} & V_C < -V_{\mathrm{BP}} \\ -\dfrac{V_C}{V_{\mathrm{BP}}} & -V_{\mathrm{BP}} \le V_C \le V_{\mathrm{BP}} \\ \dfrac{V_C - V_0}{V_0 - V_{\mathrm{BP}}} & V_C > V_{\mathrm{BP}} \end{array} \right\} \tag{2}$$

where $\pm V_0$ are the zero-crossing points of the nonlinear current, $\pm V_{\mathrm{BP}}$ are the breakpoint voltages and $\pm I_{\mathrm{BP}}$ are the breakpoint currents. Note that $-r = -V_{\mathrm{BP}}/I_{\mathrm{BP}}$.

Introducing the variables

$$X = V_C/V_{\mathrm{BP}}, \quad Y = rI_L/V_{\mathrm{BP}}, \quad \tau = t/\sqrt{LC}, \quad \alpha = r_L/r, \quad \beta = \sqrt{L/C}/r$$

and $m = V_0/V_{\mathrm{BP}}$, the dimensionless form of the above equations becomes

$$\dot{X} = -\beta(Y + f(X)) \tag{3a}$$

$$\dot{Y} = \frac{1}{\beta}(X - \alpha Y) \tag{3b}$$

$$f(X) = \left\{ \begin{array}{cc} \dfrac{X+m}{m-1} & X < -1 \\ -X & -1 \le X \le 1 \\ \dfrac{X-m}{m-1} & X > 1 \end{array} \right\}. \tag{3c}$$

A fixed nonlinear resistor implies that m is fixed. Hence, α and β control the behaviour of the oscillator. In particular, according to linear analysis, the condition for oscillation is that $\alpha \le \beta^2$. However, when this condition is satisfied, the circuit will *not* always oscillate. We demonstrate this by setting $m = 2$ and numerically integrating (3) with three different sets of α and β: $(\alpha, \beta) = (0.01, 0.1)$, $(0.99, 1)$ and $(1.1, 1.5)$. In figure 4, the time waveforms of the X and Y state variables are shown for each case. It is clear that latch up occurs when $\alpha \ge 1$.

Equations (3) can be rewritten as follows:

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} = \begin{pmatrix} -\beta a & -\beta \\ 1/\beta & -\alpha/\beta \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} -\beta b \\ 0 \end{pmatrix} \tag{4a}$$

$$(a, b) = \left\{ \begin{array}{cc} \left(\dfrac{1}{m-1}, \dfrac{m}{m-1} \right) & X < -1 \\ (-1, 0) & -1 \le X \le 1 \\ \left(\dfrac{1}{m-1}, \dfrac{-m}{m-1} \right) & X > 1 \end{array} \right\}. \tag{4b}$$

The three equilibrium points are thus given by: $(x_0, y_0) = [-b/(1 + a\alpha)](\alpha, 1)$. Hence, the origin is a *real* equilibrium point in the region $-1 \le X \le 1$. Its two eigenvalues are given by $(1/2\beta)[\beta^2 - \alpha \pm \sqrt{(\alpha - \beta^2)^2 - 4(1-\alpha)\beta^2}]$ which are complex and lie in the right-half plane if $\alpha < \beta^2$ and $\alpha < 1$. For $\alpha > 1$, both eigenvalues are real and one of them is positive if $\alpha < \beta^2$.

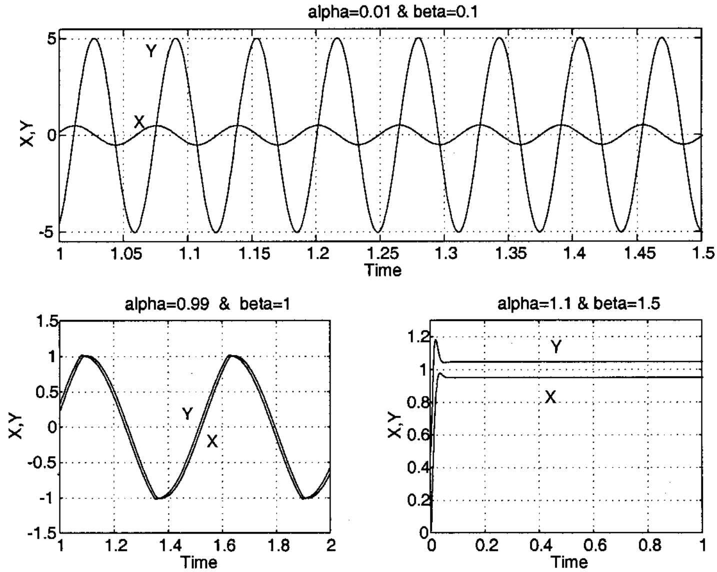

Figure 4. Numerical simulation results from the active tank resonator model for different values of α and β. Latch-up occurs for $\alpha > 1$.

In the outer regions, the equilibrium points are seen to be $(x_0, y_0) = [\mp m/(m - 1 + \alpha)](\alpha, 1)$ respectively. Hence, these equilibrium points are *virtual* for $\alpha < 1$. The critical value is $\alpha = 1$, after which these two equilibrium points become *real*, meaning that they physically lie inside the regions $X < -1$ and $X > 1$ respectively. The eigenvalues at these two equilibria are always in the left-half plane for $m > 1$. Therefore, once the system reaches any of these equilibrium points, it will settle there indefinitely since they are both *real* and *stable*, causing latch-up. Reaching one of these points is guaranteed by the positive real eigenvalue at the origin, admitted for $\alpha > 1$. Whether positive or negative latch-up will occur is totally dependent on the initial conditions on X and/or Y. In conclusion, the two proposed necessary and sufficient conditions for latch-up are satisfied for $\alpha > 1$.

It is worth noting that sinusoidal oscillators based on activated tank resonators are still widely used in RF engineering (Soyuer *et al.* 1996).

4. Less-than-unity gain oscillator

An example of an oscillator that can produce a sinusoidal waveform with less than unity gain is shown in figure 5 (Budak 1978). With equal resistors and capacitors, the characteristic equation is given by

$$(RCs)^3 + 6(1 - K)(RCs)^2 + 5(1 - K)RCs + 1 - K \tag{5}$$

which has roots on the imaginary axis when $K = 29/30$. The oscillation frequency is then $\omega_0 = 1/\sqrt{6}RC$.

In practice, it is extremely difficult to satisfy this oscillation condition since it requires a very narrow range of gain ($29/30 < K < 1$). Once K equals 1, this oscillator exhibits latch-up. To understand the reason for this behaviour, we derive a

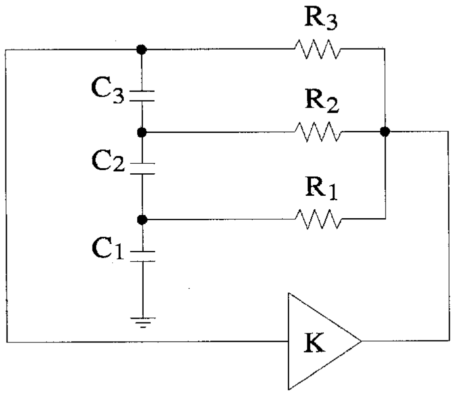

Figure 5. Sinusoidal oscillator with gain less than unity.

model taking into consideration the saturation-type characteristic of the noninverting amplifier. Thus, the oscillator is modelled by

$$C_1 \dot{V}_{C1} = \frac{V_N - V_{C1}}{R_1} + C_2 \dot{V}_{C2} \tag{6a}$$

$$C_2 \dot{V}_{C2} = \frac{V_N - V_{C1} - V_{C2}}{R_2} + C_3 \dot{V}_{C3} \tag{6b}$$

$$C_3 \dot{V}_{C3} = \frac{V_N - V_{C1} - V_{C2} - V_{C3}}{R_3} \tag{6c}$$

where V_N is the output voltage of the amplifier, given by

$$V_N = \left\{ \begin{array}{cc} V_{\mathrm{sat}} & K(V_{C1} + V_{C2} + V_{C3}) > V_{\mathrm{sat}} \\ K(V_{C1} + V_{C2} + V_{C3}) & -V_{\mathrm{sat}} \le K(V_{C1} + V_{C2} + V_{C3}) \le V_{\mathrm{sat}} \\ -V_{\mathrm{sat}} & K(V_{C1} + V_{C2} + V_{C3}) < -V_{\mathrm{sat}} \end{array} \right\}. \tag{7}$$

and V_{sat} is the saturation voltage of the amplifier.

Setting $X = V_{C1}/V_{\mathrm{sat}}, Y = V_{C2}/V_{\mathrm{sat}}, Z = V_{C3}/V_{\mathrm{sat}}$, and for the choice of $R_1 = R_2 = R_3 = R$ and $C_1 = C_2 = C_3 = C$, the dimensionless form of the above model becomes

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} 3b-3 & 3b-2 & 3b-1 \\ 2b-2 & 2b-2 & 2b-1 \\ b-1 & b-1 & b-1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} 3a \\ 2a \\ a \end{pmatrix} \tag{8a}$$

$$(a, b) = \left\{ \begin{array}{cc} (1, 0) & K(X + Y + Z) > 1 \\ (0, K) & -1 \le K(X + Y + Z) \le 1 \\ (-1, 0) & K(X + Y + Z) < -1 \end{array} \right\}. \tag{8b}$$

Numerical integration of the above model indicates that oscillations occur in the very narrow range $29/30 < K \le 1$. For $K > 1$ latch-up occurs, as shown in figure 6 for $K = 1.1$. It is clear that the origin is a real equilibrium point in the region $-1 \le K(X + Y + Z) \le 1$. This point becomes unstable when $K = 29/30$ due to

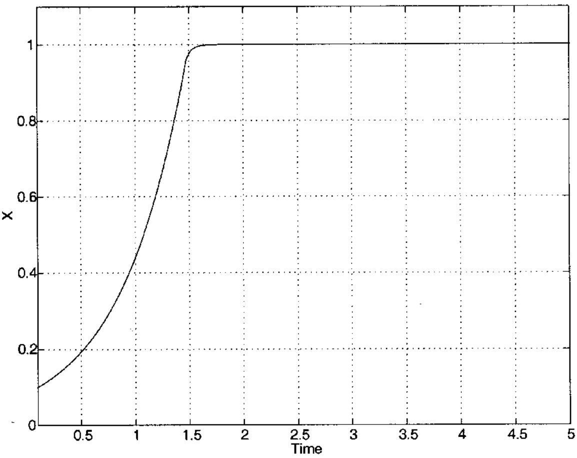

Figure 6. Numerical integration result of equation (8) showing latch-up in a less-than-unity gain oscillator with $K = 1.1$. For the equilibrium point at the origin with $K = 0.97$ the eigenvalues are $-0.196, 0.008 \pm j0.391$, and with $K = 1.1$ they are 1.124, $-0.262 \pm j0.1426$.

the appearance of a complex conjugate eigenpair. However, the additional real eigenvalue remains negative until $K = 1$, where it is zero. For $K > 1$, a positive real eigenvalue always exists at the origin.

In the outer regions $K(X + Y + Z) \gtrless \pm 1$, the equilibrium points are $(\pm 1, 0, 0)$, both of which change their state from virtual to real when $K = 1$. The eigenvalues at both points are all negative real and hence they are always stable. In conclusion, for $K > 1$ the two necessary conditions for latch-up are satisfied.

5. The Wien-bridge and phase-shift oscillators

In this section we show that the second-order Wien-bridge and third-order phase-shift oscillators cannot latch-up. Consider the Wien oscillator shown in figure 7 (*a*). Setting $X = V_{C1}/V_{\text{sat}}, Y = V_{C2}/V_{\text{sat}}$, and for the choice of $R_1 = R_2 = R$, $C_1 = C_2 = C$ with $\tau = t/RC$, this oscillator is described by

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \end{pmatrix} = \begin{pmatrix} b-2 & -1 \\ b-1 & -1 \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} + \begin{pmatrix} a \\ a \end{pmatrix} \tag{9a}$$

$$(a, b) = \left\{ \begin{array}{ll} (-1, 0) & X < -1 \\ (0, K) & -1 \le X \le 1 \\ (1, 0) & X > 1 \end{array} \right\}. \tag{9b}$$

from which it is clear that the equilibrium points are: $(x_0, y_0) = (0, a)$. Since x_0 is always zero, independent of K, the equilibrium points in the outer regions $X \gtrless \pm 1$ are always virtual and can never be real. Hence, latch-up can never occur so long as the noninverting amplifier maintains its saturation-type characteristics. We have verified this via the numerical simulation of the above model. It is worth noting that, for a particular implementation of the noninverting amplifier (eg. using a voltage op amp, OTA, etc.), the above model should be modified to include the effect of any known parasitic effect (e.g. dominant pole) (Elwakil and Soliman 1997).

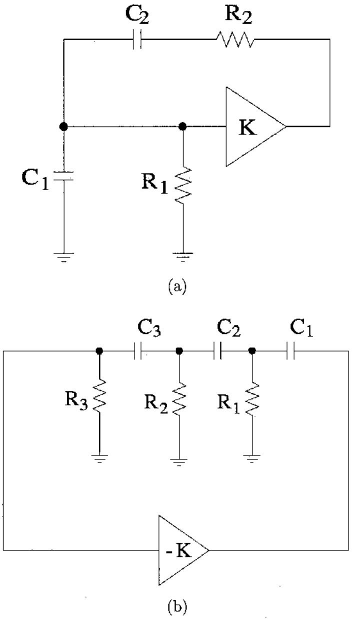

Figure 7. (*a*) Wien-bridge oscillator. (*b*) Phase-shift oscillator.

Finally, consider the phase-shift oscillator shown in figure 7 (*b*). Using the same settings as in § 4, this oscillator is described by

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = \begin{pmatrix} 3b-1 & 3b-2 & 3b-3 \\ 2b-1 & 2b-2 & 2b-2 \\ b-1 & b-1 & b-1 \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} + \begin{pmatrix} 3a \\ 2a \\ a \end{pmatrix} \tag{10a}$$

$$(a,b) = \left\{ \begin{array}{ll} (1,0) & K^*(X+Y+Z) > 1 \\ (0,K^*) & -1 \le K(X+Y+Z) \le 1 \\ (-1,0) & K^*(X+Y+Z) < -1 \end{array} \right\} \tag{10b}$$

where $K^* = K/K+1$ $(K^* < 1)$. Note that the amplifier here is inverting.

In the two outer regions $K^*(X+Y+Z) \gtrless 1$ the equilibrium points are $(\pm 1, 0, 0)$ respectively. These equilibrium points are real only if $K^* > 1$, which is impossible. Therefore, the phase-shift oscillator cannot latch-up. It is worth noting that, by applying the $RC-CR$ transformation to both the phase-shift and less-than-unity gain oscillators, the resulting counterparts exhibit the same behaviour. It is also worth noting that, for sinusoidal oscillators that are based on integrators, a similar modelling procedure can be followed†.

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† A voltage integrator can be modelled by a current amplifier with gain $\pm K$ driving a capacitive load. Hence, $V_C = I_O/sC = \pm K I_i/sC$. If the input terminal of the current amplifier is virtually grounded, then $I_i = V_i/R$, where R is an arbitrary input resistance. Therefore, $V_O/V_i = \pm K/sRC$. Oscillators that employ integrators are amplitude limited by the saturation-type characteristic of the current amplifier, similar to the case of an active tank resonator with a current-controlled negative resistor.

6. Conclusion

We have proposed two necessary and sufficient conditions for latch-up to occur in sinusoidal oscillators. We emphasize the extreme importance of accurately modelling electronic circuits and particularly oscillators as nonlinear dynamical systems (Ogorzalek 2000). If latch-up is admitted by the circuit-independent mathematical model then it cannot be avoided in any circuit-specific implementation.

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